

CFD results for TU-154M in landing configuration for an asymmetrical loss in wing length.

Summary:

In CFD work produced by G. Kowaleczko (GK) and sent to the author of this report in 2013 [1], GK concludes the total loss in lift related to the loss of a wing tip length as claimed by MAK is more than **14%** of the lifting force of the intact wing. This value is later reduced by GK to **12.5%** [6] :“*REKONSTRUKCJA OSTATNIEJ FAZY LOTU SAMOLOTU TU-154M*”. Prof. Dr hab. inż. Grzegorz Kowaleczko, 31-dec-2013. (Published 04.01.2014).

The work presented in this report covers a detailed analyze of GK's data and CFD model. From this it is demonstrated, that the CFD results of GK do not resemble the conditions of the P101 in landing configuration. The differences being of such an extent, that one should be very cautious when drawing conclusions based on the CFD results of GK with respect to the P101 case.

It can also be concluded, that the results obtained tend to clearly overestimate the lifting loss and rolling moment associated with the lost wing tip as claimed by MAK.

It is the author's view that by correcting for the main differences between the actual plane configuration and that used by GK in his CFD work, this will lead to an estimated lift loss associated with a loss of 5.54 m wingtip well below **9%**.

NOTE:

As earlier demonstrated [7] this will have an impact on the encountered roll angle following the loss of the wing tip. Even by a lift loss of 12.5% the recorded roll angle as pr the black box is left unexplained by [6]. With a lift loss of say 9% the difference between the calculated roll angles and the recorded ditto will be even more severe and even more difficult to explain.

Author: Glenn A. Jørgensen. 20-12-2013 updated 06-feb-2014

Feb 06-2014 Rev H. This update includes information of the coupling between tail angle and flaps setting, and also included the change of GK's estimated lift loss from earlier stated more than 14% (see appendix 5, page 33) to 12.5% as now claimed in [6].

BASIC CFD MODEL.

The basic CFD data of this report is based on the work of [1] (see appendix 5), in which the used model for this is claimed to be “Kubrynski_Tu-154”. The CFD calculations are performed using the software KK-aero v. 5.2 developed by K. Kubrynski. Two different configurations are calculated, plane without flap, and plane with flap each calculated for two cases: intact wing and wing loss of 6.5m . The case “with flap” is not specified in detail, but the data and 3D model point towards a flap angle of 28° as clarified in a following chapter. As seen in the following figures the 3D model is without the tail. The effect of the tail with respect to the overall lifting coefficient is explained in this work for the purpose of comparison with the overall lifting coefficient available to the public [2], [3]. The comparison clearly confirms the findings of this report.

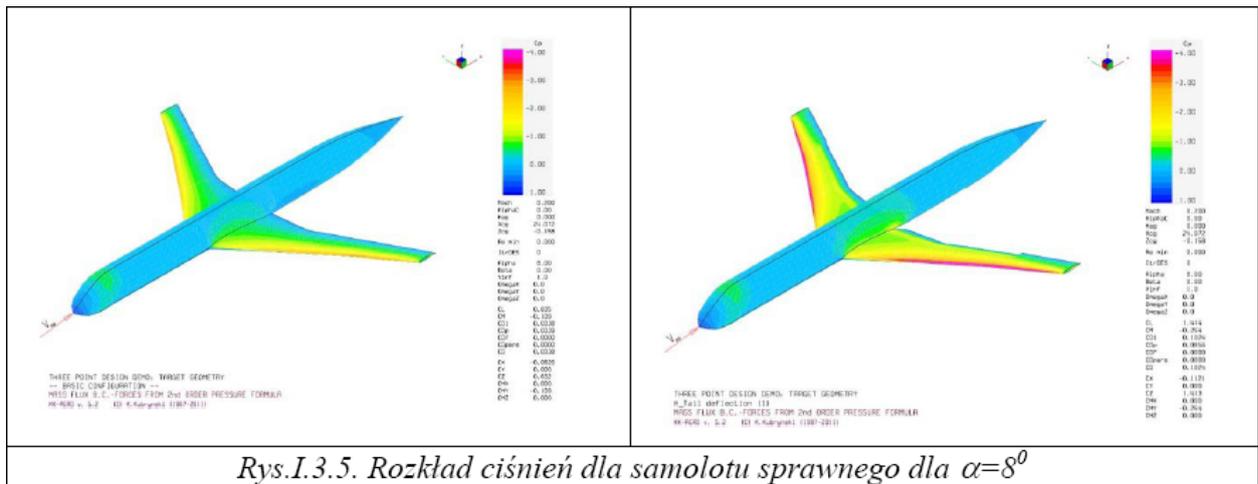


Fig. 1 The model “Kubrynski_Tu-154” with intact wing without flap (left) and with flap (right)

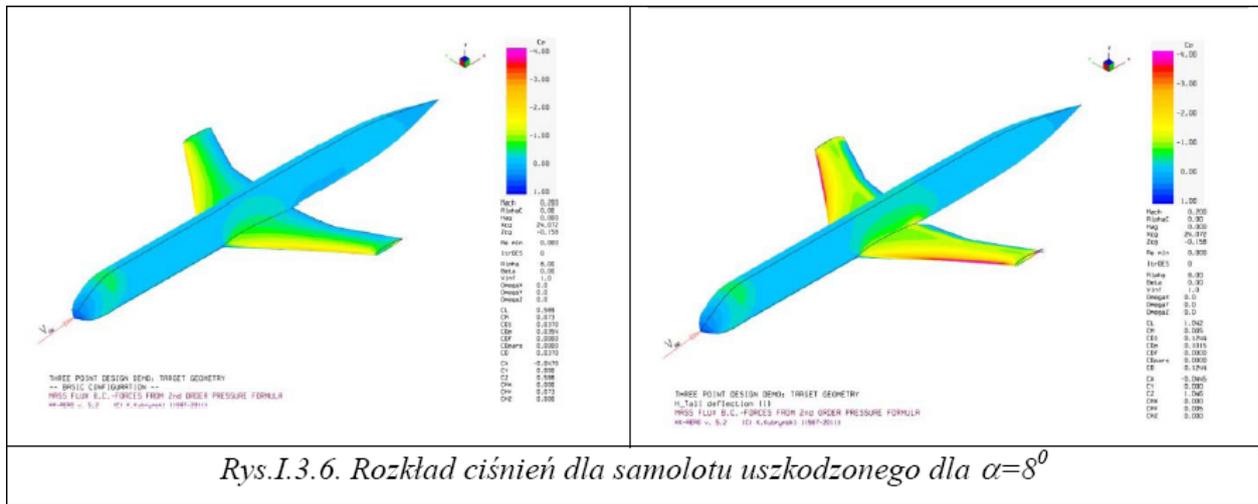


Fig. 2 The model “Kubrynski_Tu-154” with a 6.5m wing loss on both sides without flap (left) and with flap (right). Due to symmetrical restrictions in the CFD software (lack of capacity), both sides of the wing have been shortened allowing for a symmetric approach and thereby reducing the requirements of the system by 50%.

BASIC MODEL RESULTS.

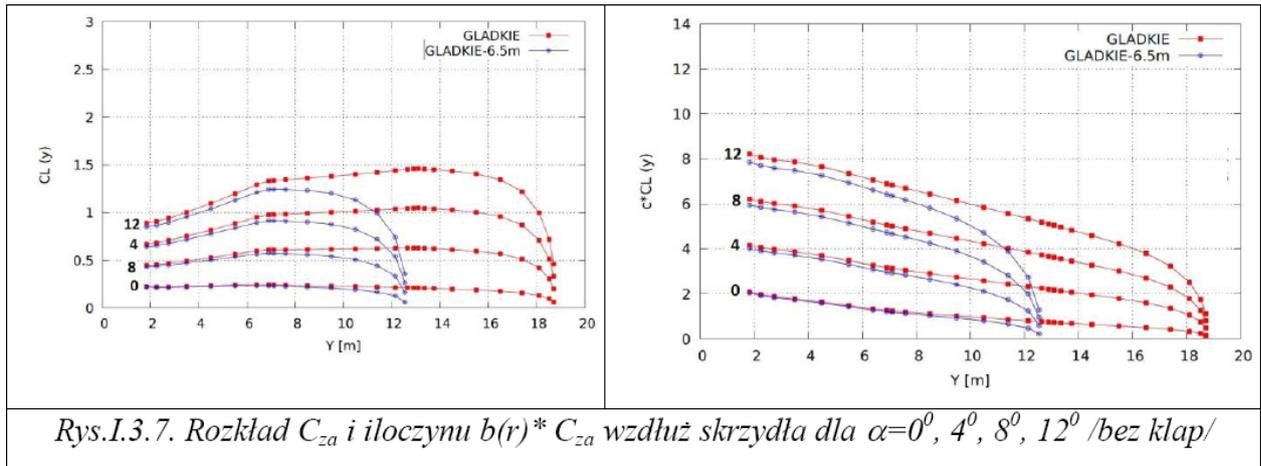


Fig. 3 The calculated lifting coefficient, CL , (left) and chord times CL , c^*CL , (right) versus wing span coordinate y for the “no flap” configuration for intact wing (red curves) and broken wing (blue curves). Original data calculated by [1], [6].

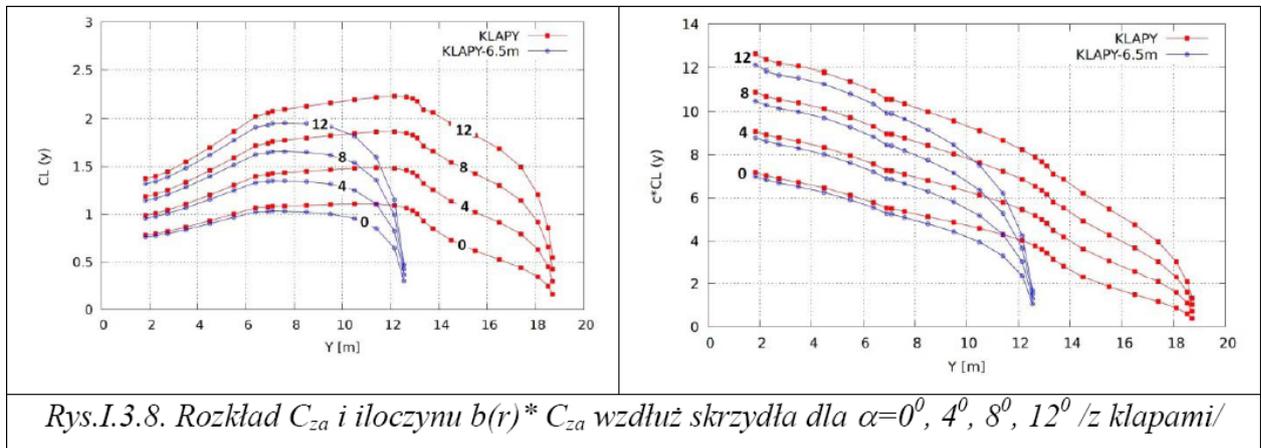
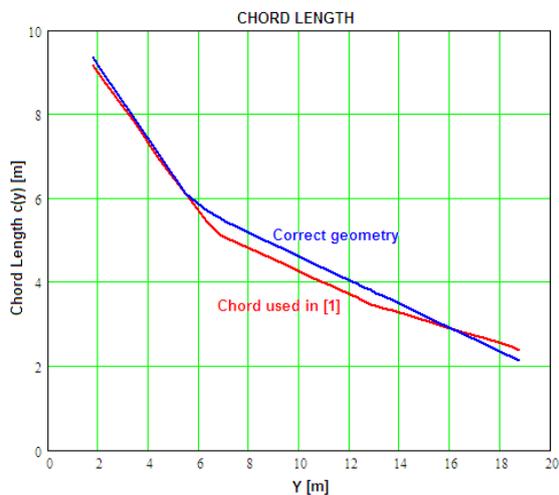


Fig. 4 The calculated lifting coefficient, CL , (left) and chord times CL , c^*CL , (right) versus wing span coordinate y for the “flap” configuration for intact wing (red curves) and broken wing (blue curves). Original data calculated by [1], [6].

Chord Length



By division of $c(y)*CL(y)$ with $CL(y)$ one can find the chord values $c(y)$ used by [1]. These values are shown together with the geometry of the wing without slats and flap as given in appendix 1. The same chord values are used for both cases with and without flap, even though the chord value increases with the extension of the double slotted flap.

Fig. 5 The $c(y)$ used in [1] (red line) is underestimated by 5% to 10% for $[6m < y < 14m]$ and exaggerated for $y > 16m$ compared to the geometry of the wing without slats and flap (blue line).

The total area outside the fuselage is found to be about $6m^2$ (or about 3%) lower than for the Tu-154M.

The overall effect of this chord incorrectness is to move relative more lift towards the wing tip section than is the case for the Tu-154M.

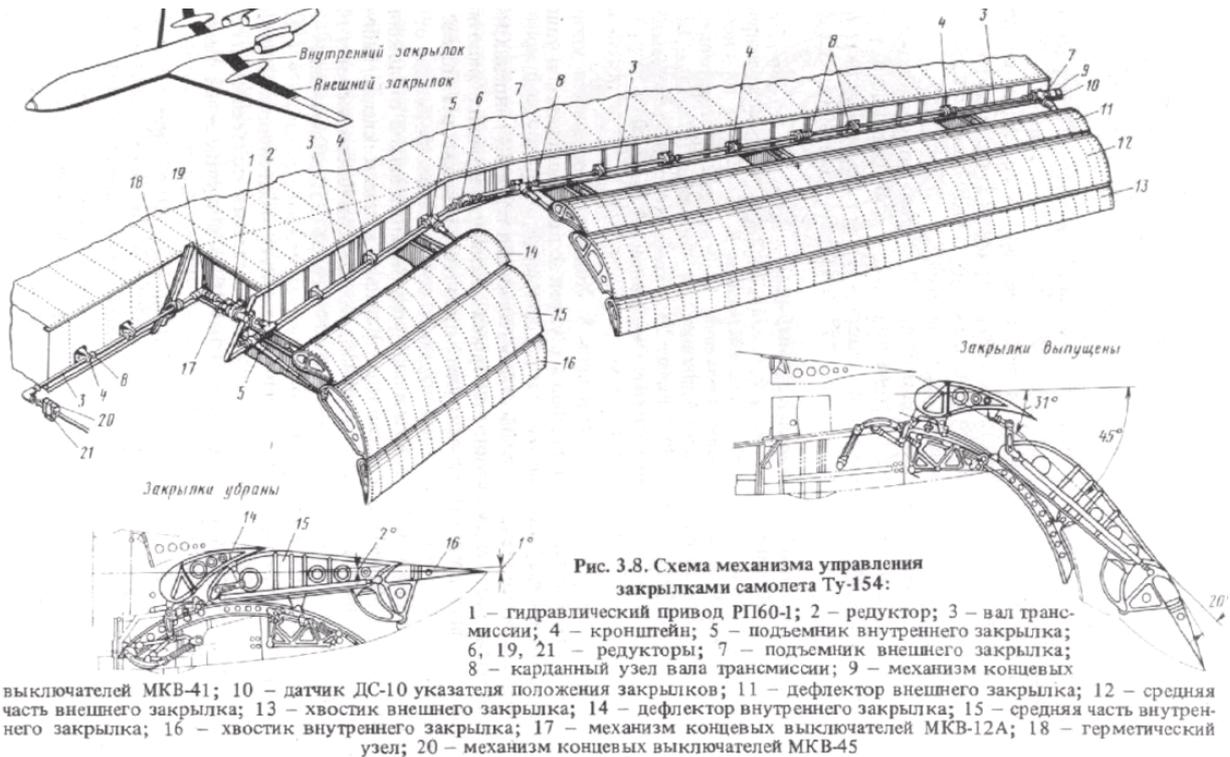


Fig. 6. The wing chord increases with the extension of flap. Figure taken from [3].

COMPARISON OF BASIC RESULTS WITH MAIN DATA OF TU-154M.

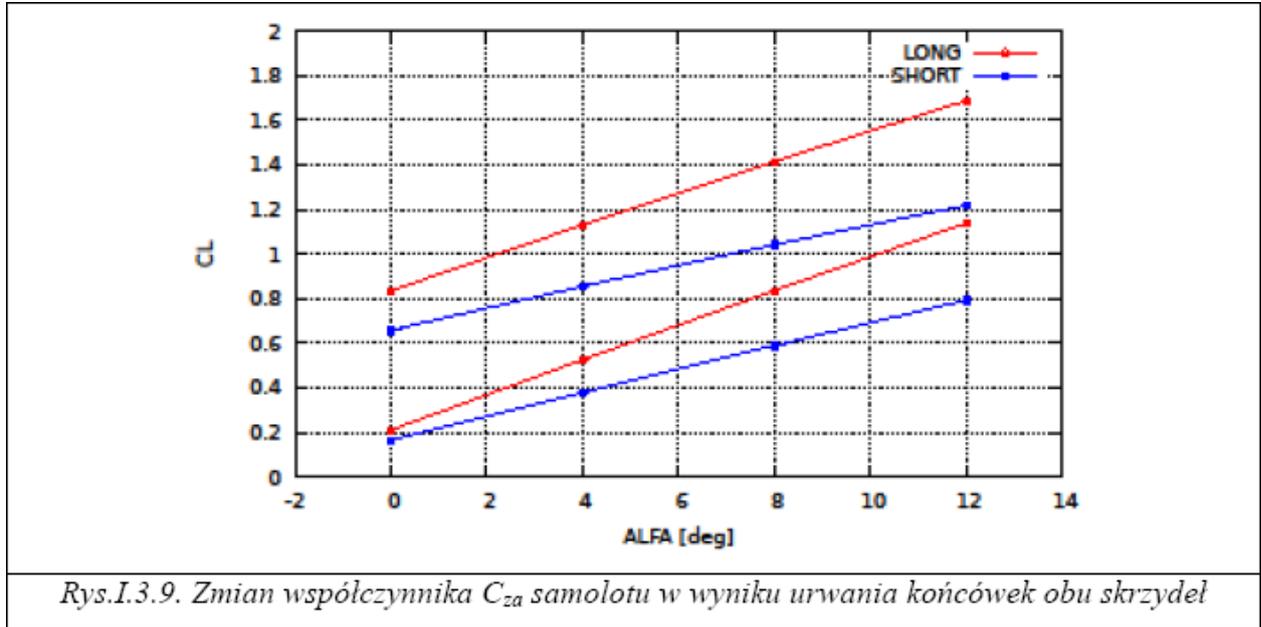


Fig. 7. The overall lifting coefficient is shown versus angle of attack for long and short wings. The figure is taken from [1], [6]. Note both sides (left and right) of the wing have been shortened, e.g. the impact is doubled as explained by GK. Note also the slope of the line of both cases of “flap” (upper two curves) is smaller than for the corresponding line of “No Flap” (Lower two curves).

The slope of the C_y found by CFD by [1] of the case “Flap” is about 8% lower than the slope of the C_y for the case of “No Flap” as seen in Fig. 7. The Tu-154M had an angle of attack of about 8° - 10°, and therefore this region has been selected for the evaluation of the distance between the two lines “With Flap” and “Without Flap”. From Fig. 9 the difference in C_y values at $\alpha=9^\circ$ is found as:

$\Delta CL(9^\circ) = 1.473 - 0.898 = 0.575$. According to [3] this difference should be $\Delta CL_{[3]} = 0.773$.

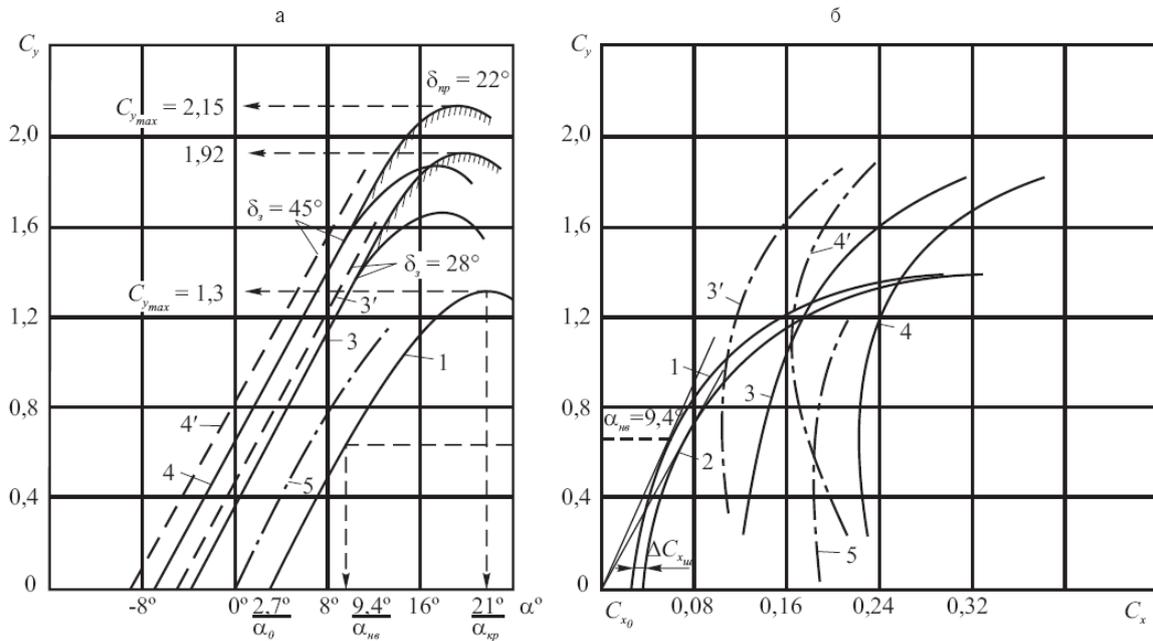


Fig. 8. The overall lifting coefficient and drag are shown versus α for the TU-154M by [2].

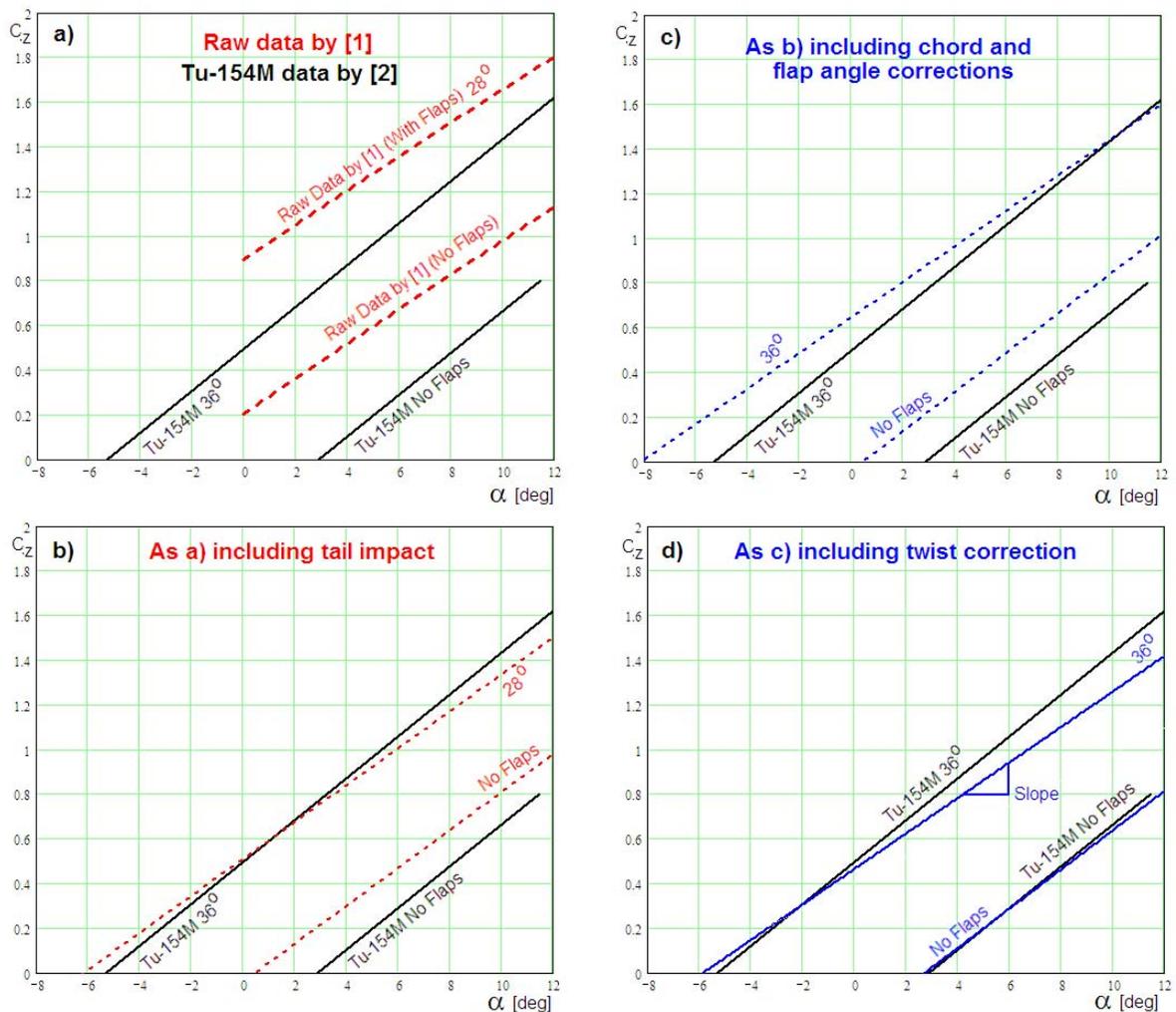


Fig. 9. The overall lifting coefficient by [2] (black lines) and a) : by [1] without inclusion of the tail impact, b) : by [1] with inclusion of the tail impact, c) : with inclusion of the tail impact, correct chord length and correct flap angle, d) including the tail impact, wing twist, correct chord length and correct flap angle as described below and in appendix 3.

As shown in **Fig. 9** the overall lifting coefficient by the work of [6] shows a fairly good correlation to that found in the russian litterature for the Tu-154M based on a reference area of $S=180\text{m}^2$. The slightly lower slope of the plain wing case without flaps is related to the smaller area of the wing used by [6] and also under influence of the other mentioned differences between the Tu154M. In case of the wing with 36° flaps, the slope found by the work of [6] is significantly lower (-12.5%) than that of the Tu-154M. The main reason for this is the use of incorrect flap type and neglecting the use of wing fences, as described in the following chapters.

Flap Angle

By Fig. 9a and Fig. 9b it is clear, that the 3D model used by [1] for the CFD calculations has insufficient amount of flap (28 deg rather than 36 deg) (see appendix 3). GK has later confirmed this observation and applied some correction [6].

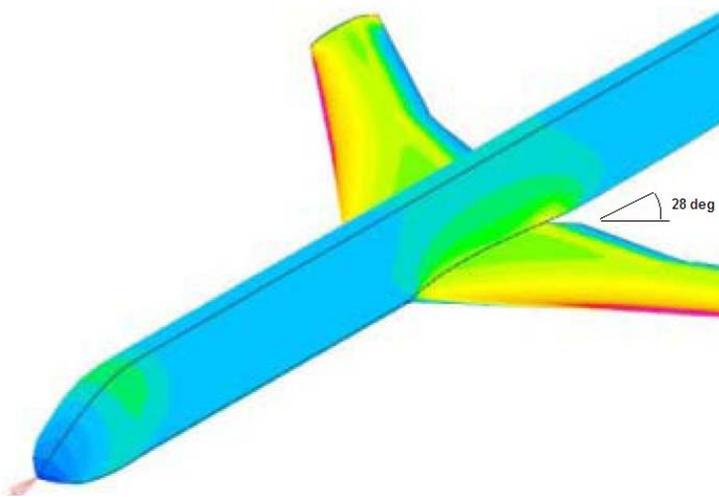


Fig. 10. The flap angle of the 3D model is seen as 28 deg. This is consistent with the analysis of the lifting distributions done in this work. GK has since confirmed this in [6] and added some correction.

Flap type.

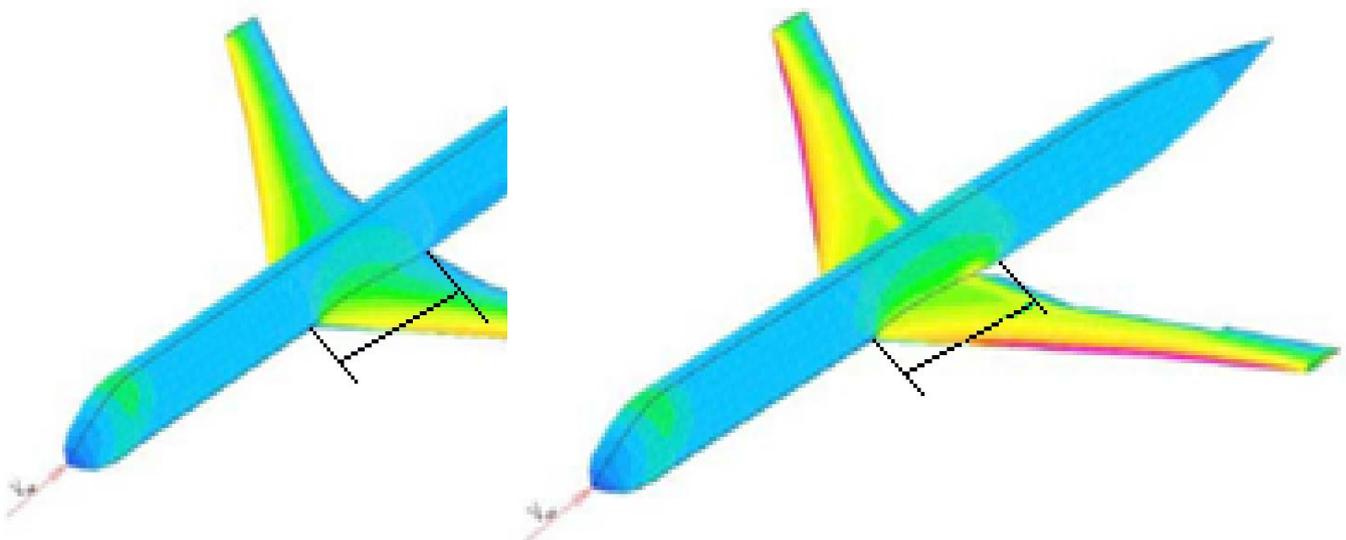


Fig. 11. The 3D model shows the flap area is built on *expense* of the normal wing area, e.g. plain hinged flaps are used. The correct flap type is double slotted fowler flaps. These have higher lift performance and are not built *in expense* of the wing area but *add* to the wing area.

High lift devices

- Other high lift devices include

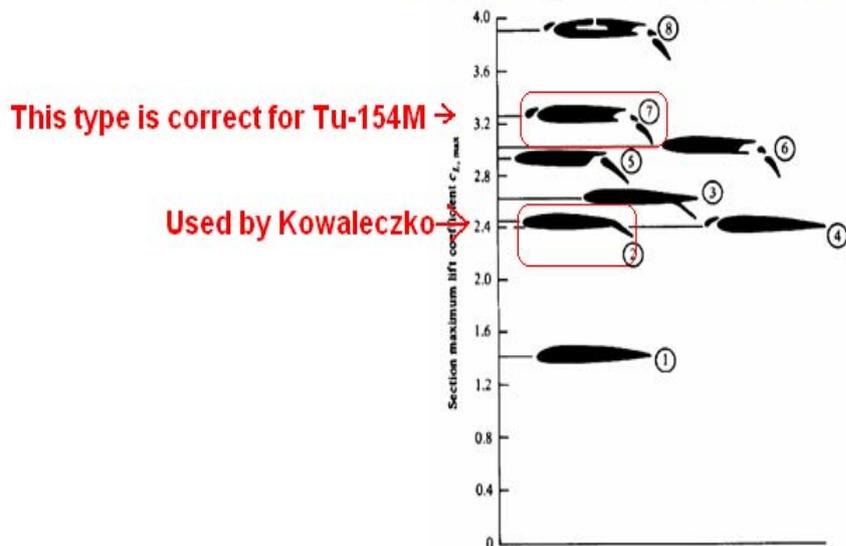


Figure 5.44 Typical values of airfoil maximum lift coefficient for various types of high-lift devices: (1) airfoil only, (2) plain flap, (3) split flap, (4) leading-edge slat, (5) single-slotted flap, (6) double-slotted flap, (7) double-slotted flap in combination with a leading-edge slat, (8) addition of boundary-layer suction at the top of the airfoil. (From Loftin, NASA SP 468, 1985.)

Fig. 12. G. Kowaleczko used a plain flap rather than a double slotted flap. According to Loftin, NASA SP 468, 1985 the maximum lift coefficient is about 35% higher for the double slotted flap in combination with a leading edge slat compared to the plain flap. Using the incorrect type of flap results in relative more lift is carried at the tip region of the wing than in case of the correct configuration, and the total lifting area is incorrect.

The 3D model used by GK in [1], [6] shows the flap area is built on *expense* of the normal wing area, e.g. plain hinged flaps are used. The correct flap type is double slotted fowler flaps. These have higher lift performance and are *not* built *in expense* of the wing area but *add* to the wing area. As seen in Fig. 12 the incorrect use of flap type will result in a less efficient lift of the wing sections encountering the flaps (inner sections close to the fuselage).

The overall effect of the incorrect flap type is to move relative more lift towards the wing tip section, rather than carrying this load by the middle and inner wing sections.

Airfoil Thickness..

Effect of airfoil thickness on lift coefficient

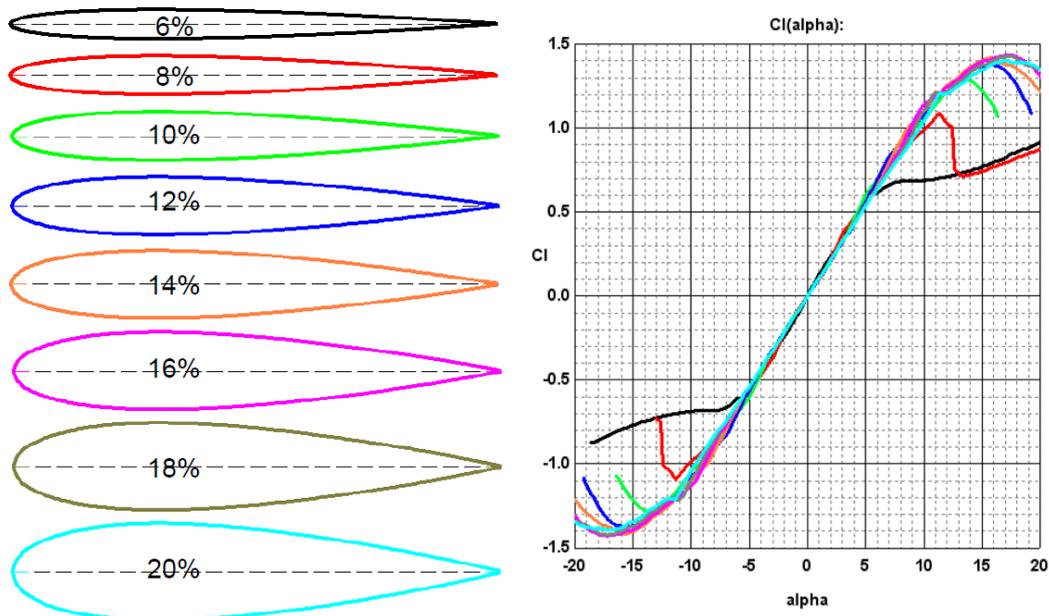


Fig. 13. This figure is borrowed from “Instytut Techniki Lotniczej i Mechaniki Stosowanej, Politechnika Warszawska” [8]. It shows the effect of profile thickness to the overall lift performance. The thick profiles tend to produce more lift for high angles of attack (α) compared to the thinner profiles.

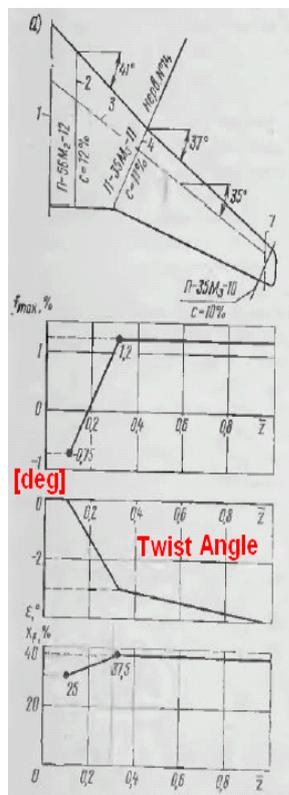


Fig. 14. The wing geometry of the TU-154M. Note the middle and tip sections of 11% and 10% thickness are slanted 35deg to the planes direction of flight.

The effective airsection thickness by [3] is 12% at the root, 9% at the middle and 8% at the tip, when taking the correct geometry into account. (Middel and tip profiles of 11% and 10%

are slanted 35deg to the XZ plane). G. Kowaleczko modelled 12% thickness over entire wing.

This is incorrect with the Tu-154M. The effective thickness is 12% at the root, 9% at the middle and 8% at the tip, when taking the correct geometry into account. (Middle and tip profiles of 11% and 10% are slanted 35deg to the XZ plane).

The result of this incorrectness of the model used by [1], [6] can be that relative more lift is carried by the wing tip section at high angles of attack than would be the case of the Tu-154M.

Wing Twist



Geometric twist

Aerodynamic twist



Fig. 15. Twist can be obtained by two means, geometric twist and aerodynamic twist. Both twist methods are present for the TU-154M. The geometrical twist seems lacking on GK's model, and as GK used the same profile throughout the entire wing GK's model has neither aerodynamic twist. Twist has a large impact on the wings lift distribution profile, and lack of this washout twist moves relative more lift towards the tip than in reality. Figure is borrowed by [8].

From Fig. 3 (left) it seems evident, that no twist is applied to the wing of the model used for CFD calculations by [1], as the local coefficient of lift shows a nearly constant behavior for increasing y except at the very outmost region where it must reach zero. The effect of twist can easily be added based on the knowledge of the dependency of $c(y)$ to α . The formulas for this operation are provided in appendix 3.

Twist has a large impact on the wings lift distribution profile, and lack of this washout twist moves relative more lift towards the tip than if it is present.

Note: The impact of several of the other discrepancies between the model used in [1],[6] and the P101 can result in the same behavior. In [6] no indication is provided by GK, to inform if the correct twist has been applied to the model used in the CFD work.

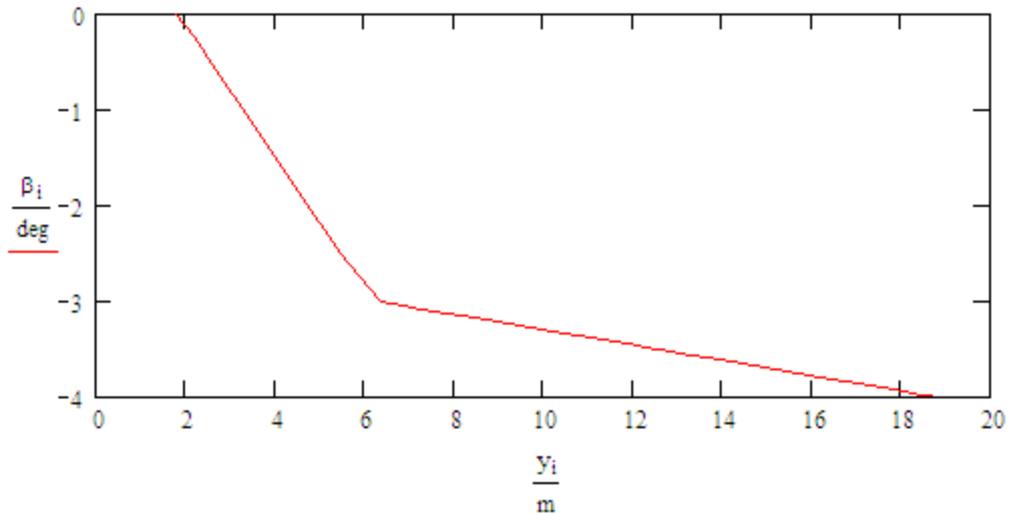


Fig. 16. The wing twist of the TU-154. Data from [3]. The tip is twisted 4 deg downwards compared to the root.

Incorrect Length of Wing Tip

In the work of [1] the length of the wing tip is stated to be $\Delta L_{\text{wing_tip}} = 6.5$ m measured in the span wise direction as written in the text of the Fig. 3 and Fig. 4, but by the curves of Fig. 3 and Fig. 4 it seems more like $\Delta L_{\text{wing_tip}} = 6.3$ m has been used. This is about 1 m more than the actual length of the left wing tip of P101, $\Delta L = 5.54$ m. (See appendix 1) By correcting this exaggeration the Lift Loss is reduced from $LF_c(8^\circ) = -0.126$ to $LF_{cc}(8^\circ) = -0.089$, or $LF_c/LF_{cc} = 1.406$ for $\alpha = 8^\circ$.

With other words the loss in lift is **25%** too high by [1], [6] caused by the mentioned exaggeration of the length of the lost wing tip.

The result of this incorrect length of the lost wing tip is to overestimate the lost lift of the wing tip and hereby overestimate the rolling moment associated with the loss of the wing tip.

Taking only this effect into account will reduce the estimated lift loss of [6] from 12.5% to 10%.

Lack of the Use of Wing Fences.



Fig. 17. The wing fences of the TU-154 shown for P101. These are not present on the model used by [1], [6].

Wing fences help prevent wing tips from stalling before the roots. They are thin plates which project up from the wing and lie parallel to the aircraft's axis of symmetry. Without these fences, a spanwise airflow along wings causes the boundary layer to thicken toward the wingtips, especially on swept-wing aircraft. This results in early boundary layer separation at the wingtips and loss of aileron control. Fences block spanwise airflow, preventing boundary layer buildup over the ailerons and thus improving stall characteristics (*USCFC*).

Wing fences are primarily found on swept-wing aircraft like business jets and fighters, but are also found on straight-winged general aviation aircraft. According to manufacturer's the airflow is redirected to the ailerons and flaps.

The result of the lack of wing fences is that relative larger amount of the lift is carried by the tip wing section, than had they been present in the model used for the CFD work.

Corrected lift distributions for some of the parameters.

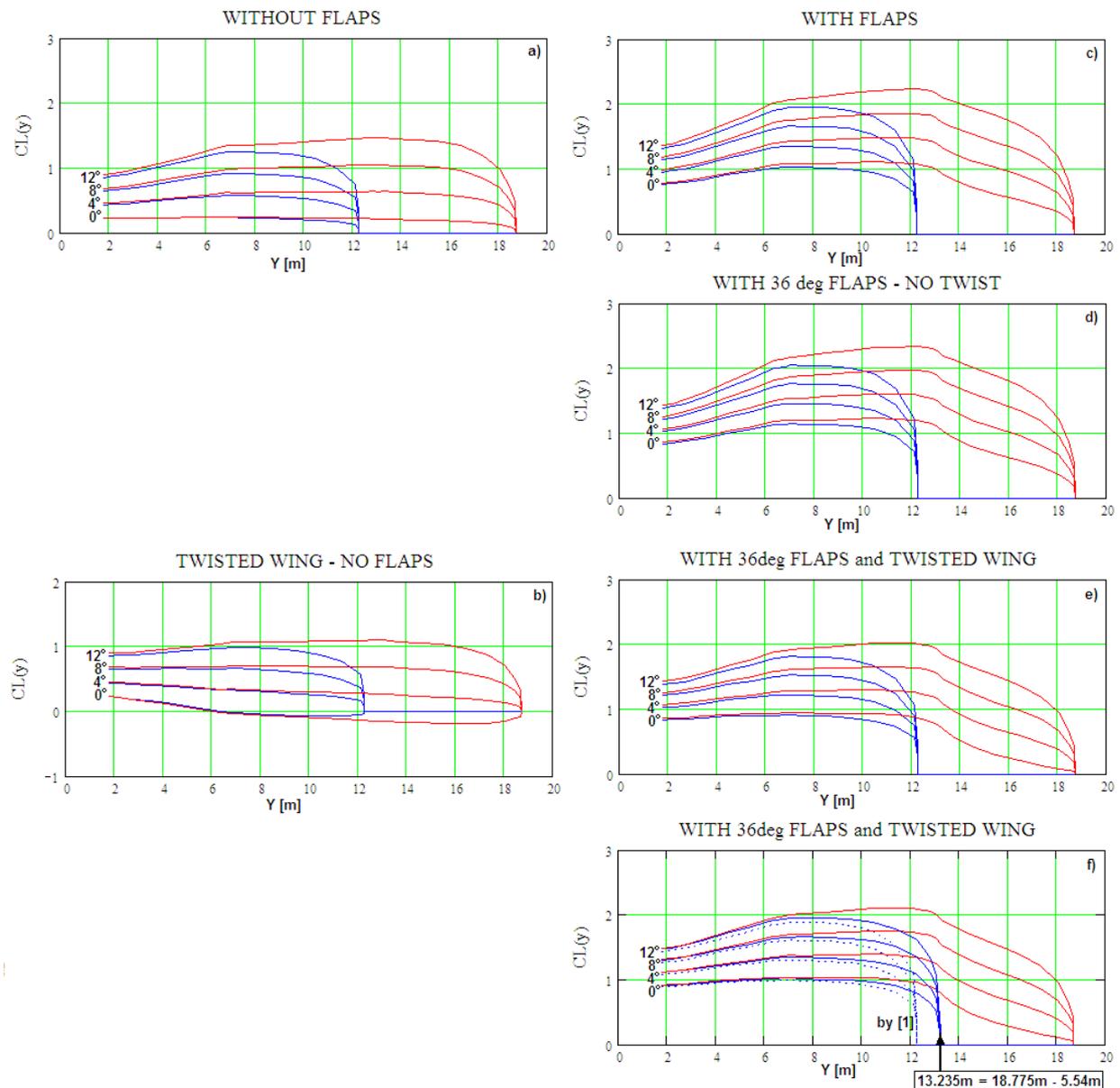


Fig. 18. The local lift coefficients $C(y)$ for the case of “no flap” (left) and “Flap” (right). The blue lines are for the broken wing and the red lines for the intact wing. Raw data taken from [1]. a) and c) : original data by [1]. b) Twisted wing without flap, d) 36 deg flap, no twist, e) 36 deg flap incl. twist, f) as e) for lost wing length of 5.54 m.

The loss ratio for loss of one wing tip is defined as :

$$LF = (CLy_b - CLy)/2CLy,$$

where CL_b is the overall lifting coefficient of the broken wing, CLy the overall lift coefficient of the intact wing. “gk” notes the raw data of [1], “c” after insertion of twist and chord length as by the wing without slats and flap. “cc” is as “c” evaluated using a wing tip length of 5.54 m. In the work described here the chord length used is as stated in appendix 1 for a wing without flap and slats.

The effect of change of flap from 28 to 36 on the lift loss is found as:

$$LF_{36} := \frac{CLF_{b36} - CLF_{36}}{2CLF_{36}} \quad LF_{36} = \begin{pmatrix} -0.114 \\ -0.13 \\ -0.14 \\ -0.148 \end{pmatrix} \quad \chi_{flaps36} := \frac{LF_{36}}{Lf_{gk}} \quad \chi_{flaps36} = \begin{pmatrix} 1.001 \\ 0.982 \\ 0.98 \\ 0.981 \end{pmatrix}$$

The effect of twist and change in flap on the lift loss is found as:

$$LF_t := \frac{CLF_{tb} - CLF_t}{2CLF_t} \quad LF_t = \begin{pmatrix} -0.085 \\ -0.11 \\ -0.126 \\ -0.137 \end{pmatrix} \quad \chi_{twist_flaps36} := \frac{LF_t}{Lf_{gk}} \quad \chi_{twist_flaps36} = \begin{pmatrix} 0.74 \\ 0.836 \\ 0.882 \\ 0.91 \end{pmatrix}$$

The effect of change in lost wing length can be found as :

$$Lf_{gk} = \begin{pmatrix} -0.114 \\ -0.132 \\ -0.143 \\ -0.151 \end{pmatrix} \quad LFc = \begin{pmatrix} -0.085 \\ -0.11 \\ -0.126 \\ -0.137 \end{pmatrix} \quad LFcc = \begin{pmatrix} -0.052 \\ -0.076 \\ -0.089 \\ -0.099 \end{pmatrix} \quad X_L := \frac{LFc}{LFcc} \quad X_L = \begin{pmatrix} 1.629 \\ 1.461 \\ 1.406 \\ 1.376 \end{pmatrix} \quad \alpha = \begin{pmatrix} 0 \\ 4 \\ 8 \\ 12 \end{pmatrix} \text{ deg}$$

The effect of change in twist can be found as :

$$\chi_{twist} := \frac{\chi_{twist_flaps36}}{\chi_{flaps36}} \quad \chi_{twist} = \begin{pmatrix} 0.739 \\ 0.851 \\ 0.9 \\ 0.927 \end{pmatrix} \quad \chi_{1en} := \frac{1}{X_L} \quad \chi_{1en} = \begin{pmatrix} 0.614 \\ 0.684 \\ 0.711 \\ 0.727 \end{pmatrix}$$

And finally the effect of change in all effects can be found as :

$$LFc := \frac{CLF_{tb} - CLF_t}{2CLF_t} \quad LFcc := \frac{cCLb - CLF_t}{2CLF_t} \quad \chi_{tot} := \frac{LFcc}{Lf_{gk}} \quad \chi_{tot} = \begin{pmatrix} 0.454 \\ 0.572 \\ 0.627 \\ 0.661 \end{pmatrix}$$

Where χ denotes the reduction factor $\chi = (\text{Actual Loss}/\text{Loss by [1]})$.

From this it is seen for $\alpha=8^\circ$, that :

$$\begin{aligned} \chi(\text{flap36}) &= 0.980 \\ \chi(\text{twist}) &= 0.900 \\ \chi(\text{twist+flap36}) &= 0.882 \\ \chi(\text{length 5.54m}) &= 0.711 \\ \chi(\text{total}) &= 0.627 \end{aligned}$$

Exaggeration = (Loss by [1])/(Actual Loss) = $1/\chi(\text{total}) = 1.59$ or an additional 59%

The total loss of lift is $LT(8^\circ) = 0.627 * 14.3\% = \mathbf{8.96\%}$

NOTE : This value is still an overestimation of the actual loss, as several important factors are not included, such as correct type of flaps, including wing fences etc.

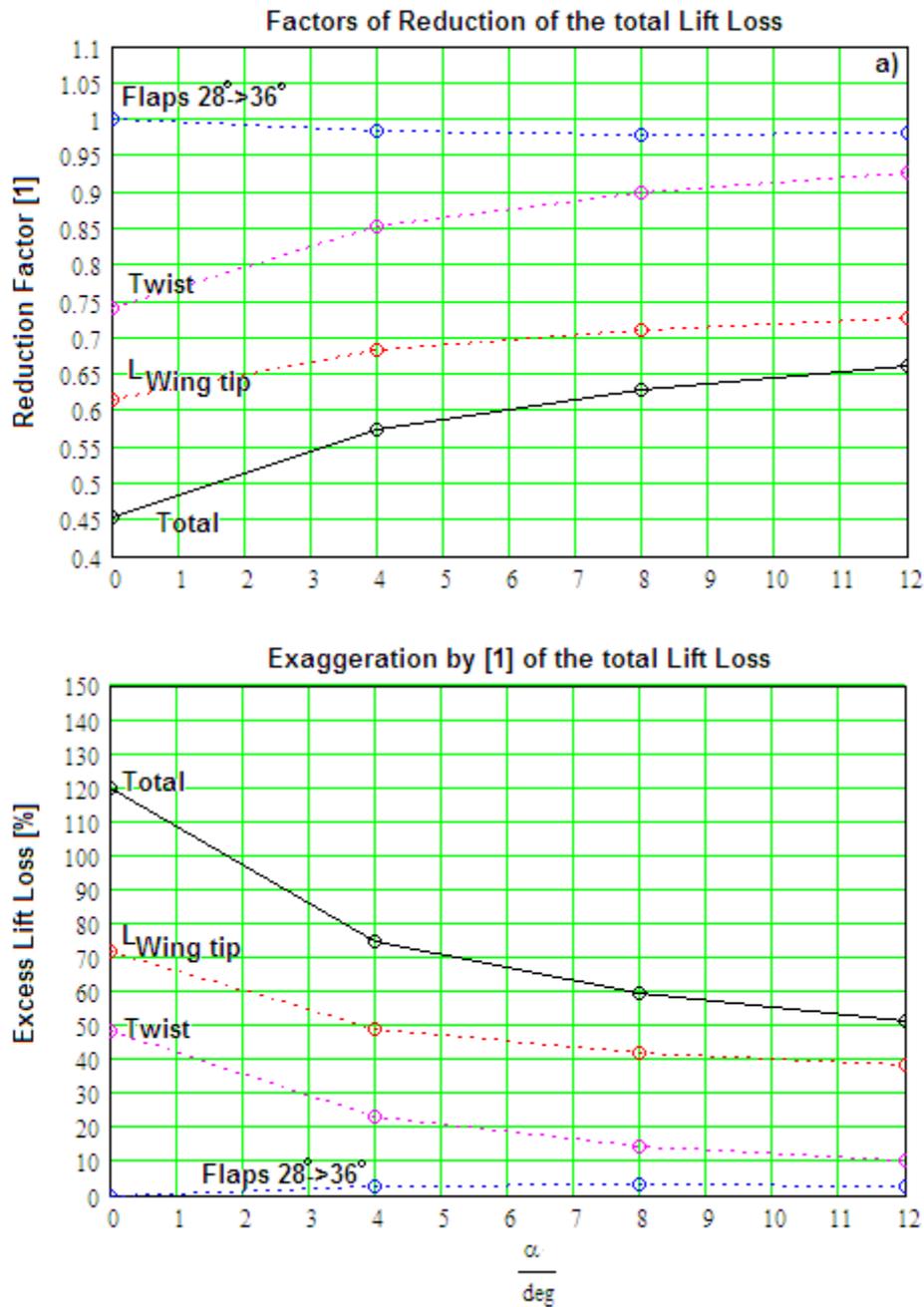


Fig. 19. **a)** The Lift Loss reduction factor associated to each component and the net product of all (black). **b)** The total excess lift loss (black) exaggerated by [1] as a function of α and divided into the contributing factors: length correction of wing tip (red), twist correction of wing (Mag) and flap correction (blue). Total exaggeration at $\alpha=8^\circ$ is 59%. NOTE these figures are based on the first claimed loss values by GK [1], and are slightly reduced with the changed estimate by GK reported in [6].

Comparison to the simple Area Method.

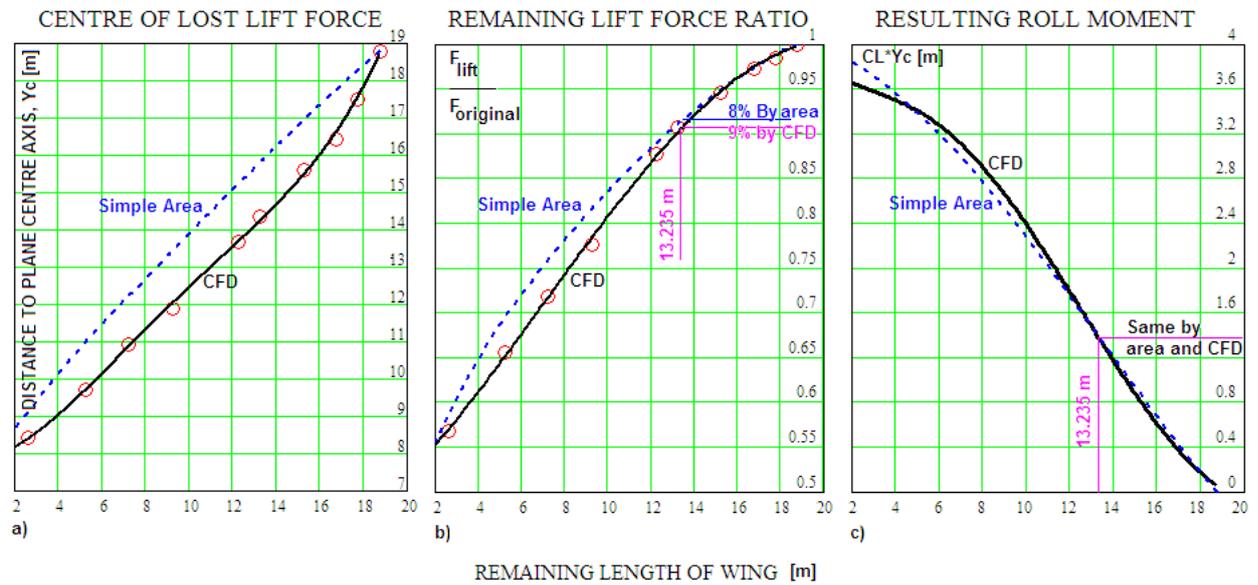


Fig. 20. Arm - Y_c a), Remaining force ratio b) and Resulting Roll Moment c) found by both the simple area based method and by CFD simulations.

As can be seen from Fig. 20 c) the resulting roll moment is very close for both methods. A CFD lift loss of 9% corresponds to a lift loss of 8% by the simple area method. **Note as the CFD estimate is found to be very conservative, the results by the simple area method can be regarded as similar conservative.**

Conclusion.

Based on Tu154M CFD Data produced by G. Kowaleczko (GK) [1],[6], GK concludes the total loss in lift related to the loss of wing tip is 12.5%% of the lifting force of the intact wing.

Detailed analyze of GK's CFD data performed in this work, show for his model the:

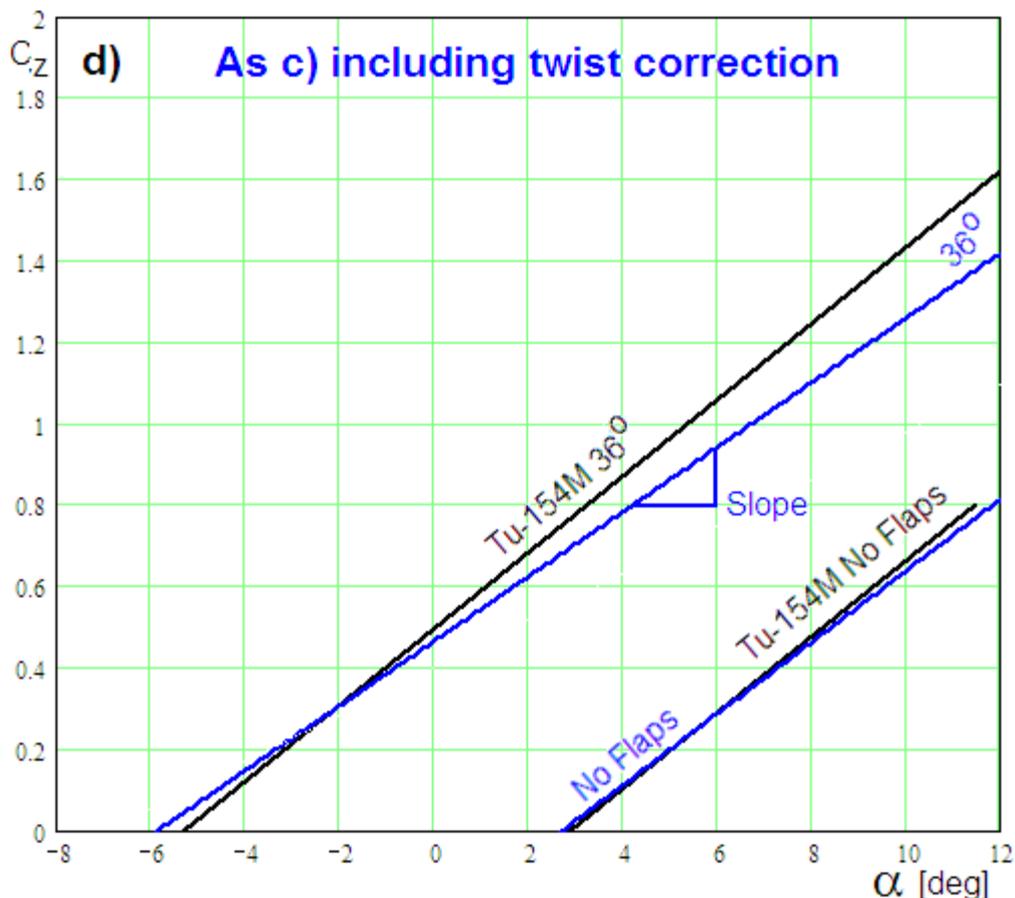
- use of incorrect length of the lost wing tip,
- distribution profiles indicate an effect similar to incorrect twist of the wing,
- incorrect wing chord lengths,
- use of incorrect flap settings (this is later handled by a correction algorithm by GK)
- use of incorrect flap type with less lifting efficiency
- use of incorrect airfoil thickness of middle and tip profiles
- lack of aerodynamic twist associated to incorrect use of airfoil sections
- lack of wing fences.

The mentioned errors each and all tend to **push more lift towards the tip section of the wing**, hereby resulting in an **over estimate** of the loss in lift caused by the loss of the wing tip. **One must be cautious to draw any conclusions about the performance of the P101 based on the CFD work of [6].**

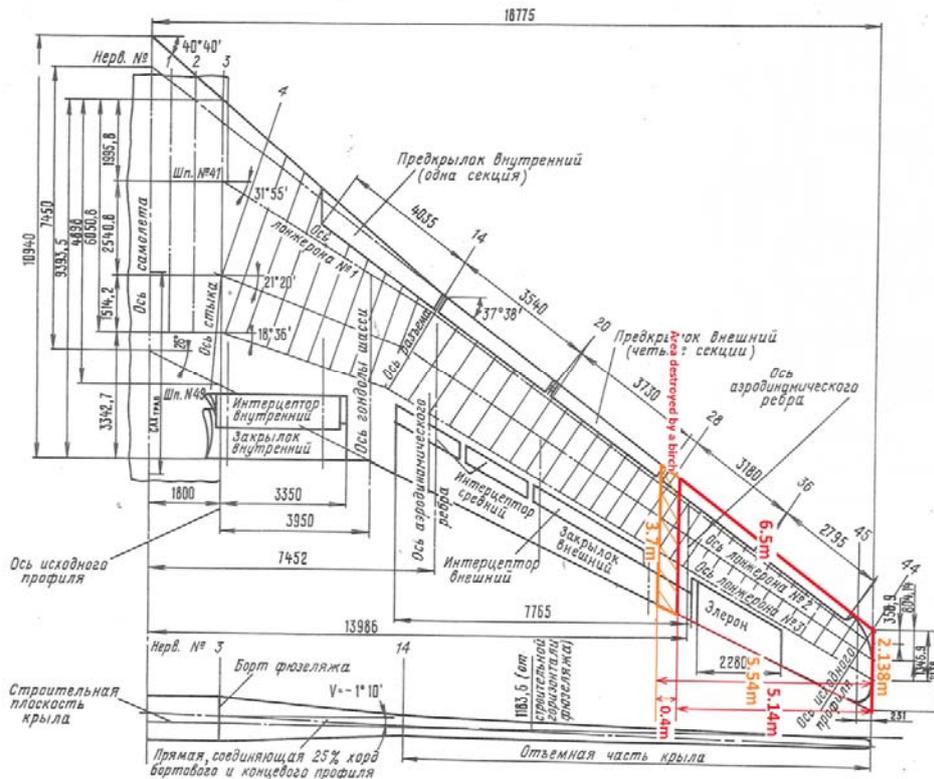
When correcting for a number (not all) of these differences with respect to the actual plane (P101), the total loss in lifting force is less than **9%** of the total lift power of the original wing.

The corrected data of the plane wing (without flaps etc) correlate fairly well with published Russian data of the TU-154M [3], when including the tail effect.

The corrected data of the wing including flap show lower lifting power for increasing angles of attack. This can be explained through the incorrect model used for the CFD work as written above, where the missing fences and the incorrect flap type errors are amongst the most severe.



APPENDIX 1. Wing Geometry.



$$\beta_0 := 19.1 \text{ deg} \quad \beta_1 := \left(37 + \frac{38}{60} \right) \cdot \text{deg} \quad \beta_2 := \left(40 + \frac{40}{60} \right) \cdot \text{deg}$$

$$L_0 := 18775 \text{ mm} \quad L_1 := 7452 \text{ mm} \quad L_2 := (1800 \text{ mm} + 3950 \text{ mm})$$

$$\Delta c_1 := L_1 \cdot (\sin(\beta_2) - \sin(\beta_1)) \quad \Delta c_1 = 0.306 \text{ m}$$

$$c_f := 10940 \text{ mm} - 7450 \text{ mm} - \Delta c_1 \quad c_f = 3.184 \text{ m}$$

$$C_{tip} := 2138 \text{ mm} \quad C_{root} := 7450 \text{ mm}$$

$$c_x(x) := \begin{cases} \Delta c_0 \leftarrow 0 \\ \Delta c_1 \leftarrow 0 \\ c \leftarrow C_{tip} + (C_{root} - C_{tip}) \cdot \frac{L_0 - x}{L_0} \\ \Delta c_0 \leftarrow \frac{L_2 - x}{L_2} \cdot c_f \text{ if } x \leq L_2 \\ \Delta c_1 \leftarrow (L_1 - x) \cdot (\sin(\beta_2) - \sin(\beta_1)) \text{ if } x \leq L_1 \\ c + \Delta c_0 + \Delta c_1 \end{cases}$$

$$c_x(L_0 - 5.54 \text{ m}) = 3.705 \text{ m}$$

APPENDIX 2. Tail Influence.

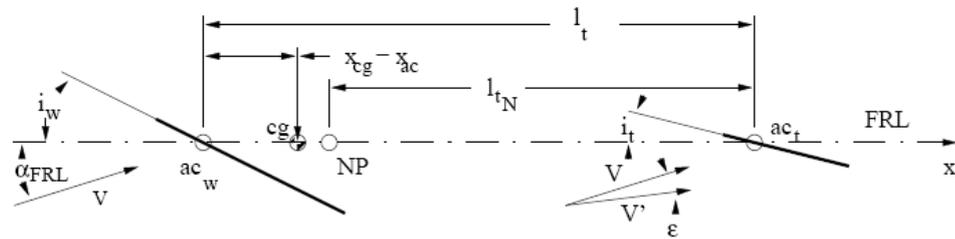


Fig. 21. Geometry of wing and tail with respect to vehicle c.g., basic neutral point, and wing aerodynamic center. Figure taken from[4].

In order to compare the data of Fig. 7 and Fig. 8 the effect of the tail on the overall lifting coefficient needs to be taken into account. By [4] the vehicle lifting coefficient, C_L , can be found as:

$$C_L = \frac{L_w + L_t}{QS} = C_{Lw} + \eta \frac{S_t}{S} C_{Lt}$$

where

η is the tail efficiency factor $\eta = Q_t/Q = (\rho V_t^2/2)/(\rho V^2/2) = V_t^2/V^2 \approx 1$ for $V_t \approx V$,

V_t is the air velocity at the tail,

V is the air velocity at the main wing,

S_t is the surface area of the horizontal tail wing,

S is the surface area of the main wing

By introducing the dependence of the lift coefficients on angle of attack and assuming a symmetrical tail section ($\alpha_{0t} = 0$) one gets [4] :

$$C_{Lw} = C_{L\alpha_w} (\alpha_{FRL} + i_w - \alpha_{0w})$$

$$C_{Lt} = C_{L\alpha_t} \left(\alpha_{FRL} + i_t - \left[\varepsilon_0 + \frac{d\varepsilon}{d\alpha} \alpha_{FRL} \right] \right)$$

and introducing these into the first equation one gets [4] :

$$C_L = C_{L\alpha_w} (i_w - \alpha_{0w}) + \eta \frac{S_t}{S} C_{L\alpha_t} (i_t - \varepsilon_0) + \left(C_{L\alpha_w} + \eta \frac{S_t}{S} \left[1 - \frac{d\varepsilon}{d\alpha} \right] C_{L\alpha_t} \right) \alpha_{FRL}$$

where

α_{FRL} is the angle of attack with reference to the fuselage reference line (FRL),

i_w is the mounting angle of the main wing root with reference to FRL,

i_t is the mounting angle of the horizontal tail wing with reference to FRL,

C_{Lw} is the overall lifting coefficient of the main wing,

C_{Lt} is the overall lifting coefficient of the horizontal tail wing,

$C_{L\alpha_w}$ is $d(C_{Lw})/d\alpha$ for the main wing

$C_{L\alpha_t}$ is $d(C_{Lt})/d\alpha$ of the horizontal tail wing,

α_{0w} is the angle of attack of zero lift for the main wing with reference to the root section.

The reduction in the angle of attack at the tail can be estimated by [4]:

$$\varepsilon = \kappa \frac{C_L}{\pi e AR}$$

where

e is the wing efficiency factor, for elliptical wings $e = 1.0$, for non-elliptical span loadings $e < 1.0$.

AR is the wing aspect ratio $AR = b^2/S$ and

b is the wing span width (37.55m)

The rate of tail downwash with angle of attack can then be estimated as [4] :

$$\frac{d\varepsilon}{d\alpha} = \frac{\kappa}{\pi e AR} (C_{L\alpha})_{wing}$$

where $1 < \kappa < 2$, and $\kappa=1$ corresponds to $\varepsilon = \alpha_i$, the induced angle of attack of the wing while $\kappa=2$ corresponds to the limit when the tail is far downstream of the wing [4].

According to [5], the induced drag coefficient can be written as :

$$C_{D,i} = \frac{C_L^2}{\pi A R e}$$

The tail angle to the fuselage reference line is variable between -3deg to -8.5deg depending upon the flap configuration and the centre of gravity setting.

From knowledge of $C_{D,i}$ found in the CFD calculations [1] one can estimate the span efficiency factors. The results are listed in Tabel 1.

Tabel 1.	S	b	AR
Span efficiency coefficients based on $C_{D,i}$ by [1], $\alpha=8^\circ$	[m ²]	[m]	[1]
Short Wing	147.8	26.55	4.77
Long Wing	180	37.55	7.83
Case	$C_{D,i}$	CL_w	e
Long Wing with No flap	0.0338	0.835	0.838
Long Wing with Flap	0.1024	1.414	0.793
Short Wing with No Flap	0.037	0.589	0.626
Short Wing with Flap	0.1244	1.042	0.583
Total lift loss with flap by raw data of [1] ($CL_{wb} - CL_w$)/(2* CL_w) , where "b" denominates "broken"		-13.15%	Not 14% as claimed by [1]

The total loss in lift based for $\alpha=8^\circ$ is found as 13.15%. (See Tabel 1).

$$e(b, S) := \frac{1}{1 + 0.09 \cdot \frac{b^2}{S}}$$

These values are close to the value given by [3] as written in the above equation for the intact wing.

As the efficiency factor is used to estimate the downwash in the tail region it is reasonable to take into account, that the CFD analysis is performed on a model with both sides of the wing shortened. Therefore the value $e = (e_0 + e_b)/2$ is more correct for the case only one side is shortened, where e_0 is the efficiency of the intact wing by tabel 1, and e_b the efficiency by tabel 1 for the double sided shortened model. The influence of this parameter is very weak, so minor errors in this parameter has only little importance.

CL_t is the overall lifting coefficient of the horizontal tail wing,

$CL_{t\alpha}$ is $d(CL_t)/d\alpha$ of the horizontal tail wing,

can be evaluated by :

$$a = \frac{\pi AR}{1 + \sqrt{1 + \left(\frac{\pi AR}{a_0 \cos \Lambda_c/4}\right)^2}}$$

$$CL_{\alpha t}(a_0, S, b, \Lambda_{c025}) := \frac{\pi \cdot \left(\frac{b^2}{S}\right)}{1 + \sqrt{1 + \left[\frac{\pi \cdot \left(\frac{b^2}{S}\right)}{a_0 \cdot \cos(\Lambda_{c025})}\right]^2}}$$

For the tail one has :

$$\Lambda_{c025} := 40 \cdot \text{deg}$$

$$b_t := 13.4 \cdot \text{m}$$

$$CL_{\alpha t} := dCL d\alpha_w$$

$$CL_{\alpha t} := CL_{\alpha t}(2 \cdot \pi, S_t, b_t, \Lambda_{c025})$$

$$CL_{\alpha t} = 3.388$$

APPENDIX 3. Addition of Twist and Correction of Flap Angle.

$$\text{eff}(f) := -0.0001 \cdot \left(\frac{f}{\text{deg}} \right)^2 + 0.0274 \cdot \left(\frac{f}{\text{deg}} \right)$$

$$\zeta := \frac{\text{eff}(36 \cdot \text{deg})}{\text{eff}(28 \cdot \text{deg})}$$

$$\zeta = 1.244$$

$$\text{Cfb}_{36_0} := (\text{Cfb}_0 - \text{Cb}_0) \cdot \zeta + \text{Cb}_0$$

$$\text{Cf}_{36_0} := (\text{Cf}_0 - \text{C}_0) \cdot \zeta + \text{C}_0$$

$$\text{Cfb}_{36_4} := (\text{Cfb}_4 - \text{Cb}_4) \cdot \zeta + \text{Cb}_4$$

$$\text{Cf}_{36_4} := (\text{Cf}_4 - \text{C}_4) \cdot \zeta + \text{C}_4$$

$$\text{Cfb}_{36_8} := (\text{Cfb}_8 - \text{Cb}_8) \cdot \zeta + \text{Cb}_8$$

$$\text{Cf}_{36_8} := (\text{Cf}_8 - \text{C}_8) \cdot \zeta + \text{C}_8$$

$$\text{Cfb}_{36_{12}} := (\text{Cfb}_{12} - \text{Cb}_{12}) \cdot \zeta + \text{Cb}_{12}$$

$$\text{Cf}_{36_{12}} := (\text{Cf}_{12} - \text{C}_{12}) \cdot \zeta + \text{C}_{12}$$

$$d\beta_f := \frac{\overrightarrow{[(\text{Cfb}_{36_{12}} - \text{Cfb}_{36_0}) \cdot \beta]}}{12 \cdot \text{deg}}$$

$$d\beta_f := \frac{\overrightarrow{[(\text{Cf}_{36_{12}} - \text{Cf}_{36_0}) \cdot \beta]}}{12 \cdot \text{deg}}$$

$$d\beta_b := \frac{\overrightarrow{[(\text{Cb}_{12} - \text{Cb}_0) \cdot \beta]}}{12 \cdot \text{deg}}$$

$$d\beta := \frac{\overrightarrow{[(\text{C}_{12} - \text{C}_0) \cdot \beta]}}{12 \cdot \text{deg}}$$

$$\text{Cf}_{36t_0} := d\beta_f + \text{Cf}_{36_0}$$

$$\text{Cfb}_{36t_0} := d\beta_b + \text{Cfb}_{36_0}$$

$$\text{Cf}_{36t_4} := d\beta_f + \text{Cf}_{36_4}$$

$$\text{Cfb}_{36t_4} := d\beta_b + \text{Cfb}_{36_4}$$

$$\text{Cf}_{36t_8} := d\beta_f + \text{Cf}_{36_8}$$

$$\text{Cfb}_{36t_8} := d\beta_b + \text{Cfb}_{36_8}$$

$$\text{Cf}_{36t_{12}} := d\beta_f + \text{Cf}_{36_{12}}$$

$$\text{Cfb}_{36t_{12}} := d\beta_b + \text{Cfb}_{36_{12}}$$

$$\text{Ct}_0 := d\beta + \text{C}_0$$

$$\text{Ctb}_0 := d\beta_b + \text{Cb}_0$$

$$\text{Ct}_4 := d\beta + \text{C}_4$$

$$\text{Ctb}_4 := d\beta_b + \text{Cb}_4$$

$$\text{Ct}_8 := d\beta + \text{C}_8$$

$$\text{Ctb}_8 := d\beta_b + \text{Cb}_8$$

$$\text{Ct}_{12} := d\beta + \text{C}_{12}$$

$$\text{Ctb}_{12} := d\beta_b + \text{Cb}_{12}$$

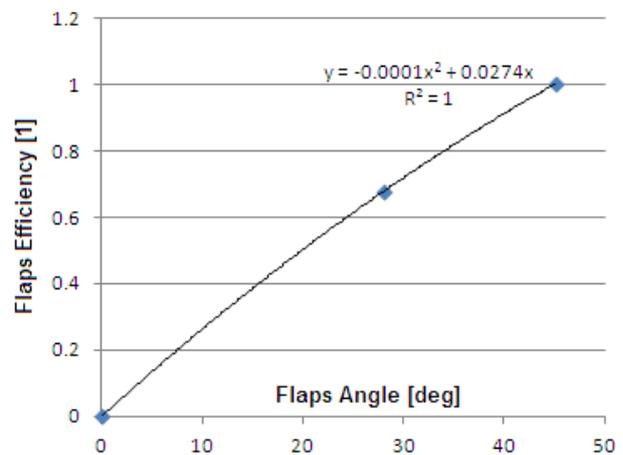
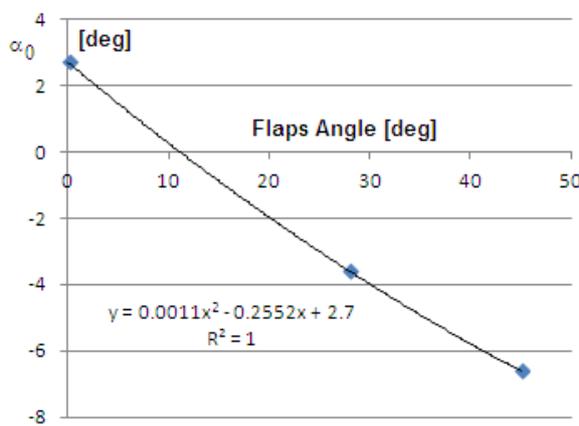
Where

- Cfb_x is the local coefficient of lift for the broken wing at $\alpha = x$
- Cfb_{36_x} is local coefficient of lift for the broken wing and flap angle = 36° at $\alpha = x$
- Cf_{36_x} as above including twist at $\alpha = x$
- β is the twist vector as defined in [3]. See Fig. 16.
- eff(f) is the flap efficiency function. See below.

Основные аэродинамические характеристики самолета Ту-154М при различных конфигурациях на высоте и у земли

Конфигурация самолета	α_0°	C_{x0}	$\alpha_{нв}^\circ$	K_{max}	$\alpha_{кр}^\circ$	$C_{y_{max}}$	$V_{св}$, км/ч при $m=100$ т
Полетная; шасси выпущено	2,7	0,022	9,4	16,5	21	1,3	315
	2,7	0,037	10,5	11,5	21	1,3	315
Взлетная; шасси выпущено $\delta_3=28^\circ$; $\delta_{np}=22^\circ$; на высоте	-3,6	0,031	10,6	7,0	18	1,92	235
	-4,6	-	10,0	10,8	-	-	-
Посадочная; шасси выпущено $\delta_3=45^\circ$; $\delta_{np}=22^\circ$; на высоте	-6,6	0,238	11,6	5,6	17,8	2,15	230
	-9,0	-	11,5	8,7	-	-	-

Data from [2] show the shift in α_0 as a function of the flap angles 28° and 45° . These are used to interpolate to the value at flap angle of 36° taking the small non-linearity into account.



The flap efficiency E_f is defined as

$$E_f = \frac{\alpha_0(\delta_3) - \alpha_0(0\text{-deg})}{\alpha_0(45\text{-deg}) - \alpha_0(0\text{-deg})} = \frac{\alpha_0(\delta_3) - 2.7\text{-deg}}{-6.6\text{-deg} - 2.7\text{-deg}} = \frac{2.7\text{-deg} - \alpha_0(\delta_3)}{9.3\text{-deg}}$$

Where $\delta_3 = \delta$ is the flap angle and α_0 is the angle of attack for zero lift as listed in the table above.
 $E_f(\delta) = -0.0001 \cdot \delta^2 + 0.0274 \cdot \delta$

This functionality takes the small non-linearity of the shift in α_0 as a function of flap angle into account. From this the effect of change of flap from 28° to 36° can be found as:

$$E_f(36^\circ) / E_f(28^\circ) = 1.244$$

APPENDIX 4. Correction of wing tip length.

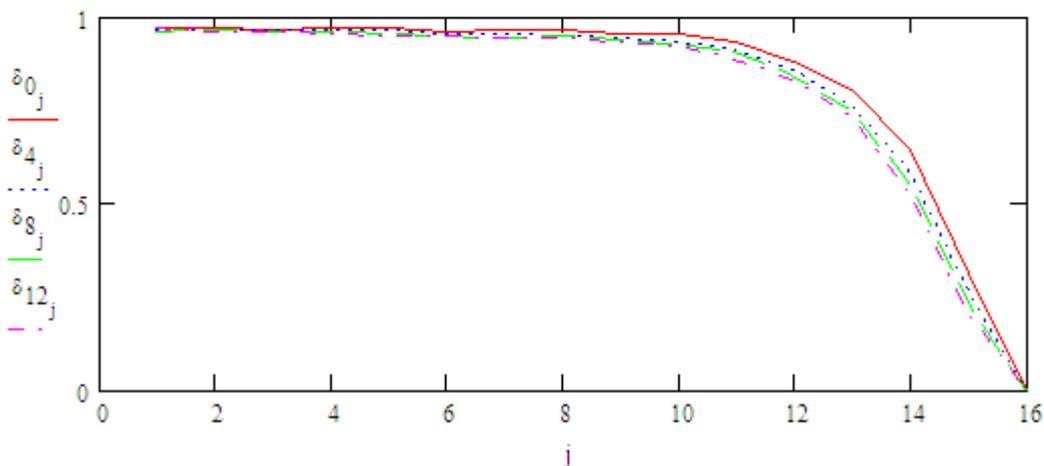
$$\Delta x_{wt} := 0 \cdot m \quad x := \frac{\Delta x_{wt}}{\Delta x_{wt_GK}}$$

$$r_0 := \frac{Cf_{36t_0_0}}{Cfb_{36t_0_0}} \quad r_4 := \frac{Cf_{36t_4_0}}{Cfb_{36t_4_0}} \quad r_8 := \frac{Cf_{36t_8_0}}{Cfb_{36t_8_0}} \quad r_{12} := \frac{Cf_{36t_{12}_4}}{Cfb_{36t_{12}_4}}$$

$$\eta_0 := 1 + (r_0 - 1) \cdot x \quad \eta_4 := 1 + (r_4 - 1) \cdot x$$

$$\eta_8 := 1 + (r_8 - 1) \cdot x \quad \eta_{12} := 1 + (r_{12} - 1) \cdot x$$

$$\delta_0 := \frac{Cfb_{36t_0}}{Cf_{36t_0}} \cdot \eta_0 \quad \delta_4 := \frac{Cfb_{36t_4}}{Cf_{36t_4}} \cdot \eta_4 \quad \delta_8 := \frac{Cfb_{36t_8}}{Cf_{36t_8}} \cdot \eta_8 \quad \delta_{12} := \frac{Cfb_{36t_{12}}}{Cf_{36t_{12}}} \cdot \eta_{12}$$



The function $r = Cfb/Cf$ shows how the local lift coefficients are influenced by the loss of wing length for $\alpha=0^\circ$ to $\alpha=12^\circ$.

When evaluating the local lift coefficients for at lost wing tip of $\Delta L = 5.54$ m the same dependency is used, but now stretched to the new position.

APPENDIX 5. Tu 154 wing aerodynamics ([1]).

3.2. Siły i momenty aerodynamiczne skrzydła

Jak zaznaczono powyżej, siły i momenty aerodynamiczne skrzydła obliczono poprzez całkowanie odpowiednich wyrażeń wzdłuż skrzydła samolotu. W tym celu dla każdego profilu skrzydła określono lokalny układ współrzędnych $P\rho\kappa\tau$. Układ ten pokazany został na rysunku I.3.1. Definiowany on jest następująco:

Początek układu położony jest w punkcie P leżącym na na $\frac{1}{4}$ cięciwy. Oś $P\rho$ pokrywa się z cięciwą i skierowana jest w kierunku noska profilu¹. Płaszczyzna $P\rho\tau$ jest płaszczyzną profilu. Oś $P\kappa$ jest prostopadła do płaszczyzny $P\rho\tau$ i skierowana w kierunku końca skrzydła.

Punkt P ma w układzie związanym z samolotem $Oxyz$ współrzędne (x_P, y_P, z_P) . Punkty P kolejnych profili skrzydła skośnego wyznaczają linię $\frac{1}{4}$ cięciw. Przyjmując y_P za współrzędną niezależną można dwie pozostałe współrzędne tego punktu zapisać jako $x_P(y_P)$ oraz $z_P(y_P)$. W ogólnym przypadku linia $\frac{1}{4}$ cięciw jest krzywą.

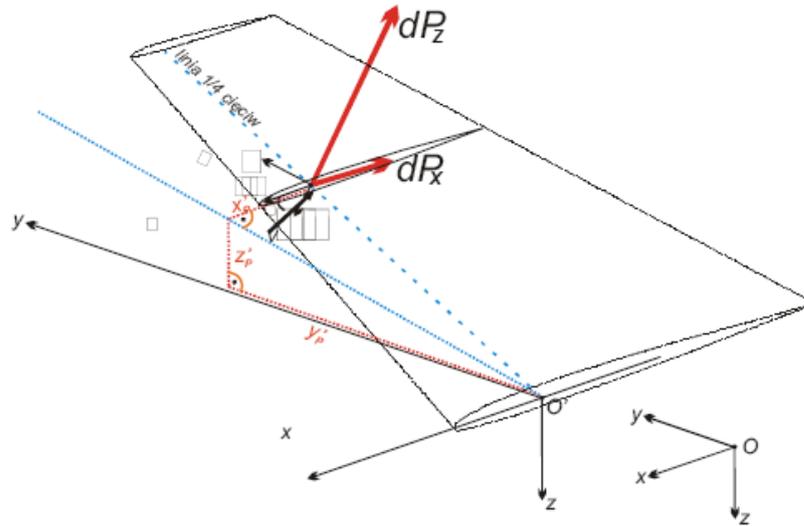
Wprowadzając pomocniczy układ współrzędnych $O'xyz$, którego osie są równoległe do osi układu $Oxyz$, zaś jego początek O' położony jest w punkcie o współrzędnych $(x_{O'}, 0, z_{O'})$ ² i zakładając, że samolot ma skrzydło o stałym skosie χ oraz wzniosie ψ położenie punktu P w układzie związanym z samolotem można opisać zależnościami:

$$x_P = x_{O'} + x'_P = x_{O'} - y_P \tan \chi_{sk} \quad (I.3.4)$$

$$z_P = z_{O'} + z'_P = z_{O'} - y_P \frac{\tan \psi_{sk}}{\cos \chi_{sk}} \quad (I.3.5)$$

¹ Jeżeli skrzydło nie ma skręcenia geometrycznego i kąta zaklinowania, to oś $P\rho$ jest równoległa do osi podłużnej samolotu Ox .

² Punkt ten leży na przecięciu linii $\frac{1}{4}$ cięciw z płaszczyzną symetrii samolotu Oxz .



Rys.I.3.1. Położenie układu związanego z profilem skrzydła względem układu $Oxyz$ oraz elementarne siły generowane przez profil

Macierz przejścia $\mathbf{L}_{s/p}$ z układu $P\rho\kappa\tau$ do układu $Oxyz$ otrzymuje się dokonując kolejnych obrotów:

- obrót o kąt skręcenia skrzydła ϕ_{sk} - sprowadza cięciwę profilu (oś $P\rho$) do płaszczyzny równoległej do płaszczyzny Oxy ;
- obrót o kąt wzniosu skrzydła ψ_{sk} - sprowadza skrzydło do płaszczyzny Oxy tzn. oś $P\kappa$ po obrocie leży w płaszczyźnie Oxy ;
- obrót o kąt skosu skrzydła χ_{sk} - sprowadza skrzydło do płaszczyzny Oxy tzn. oś $P\kappa$ po obrocie jest równoległa do osi Oy ;

Wektor, który w układzie $P\rho\kappa\tau$ ma składowe $[\rho, \kappa, \tau]$ będzie w układzie $Oxyz$ miał składowe $[x, y, z]^T$, Zachodzi następujący związek:

$$\begin{bmatrix} \rho \\ \kappa \\ \tau \end{bmatrix} = \mathbf{L}_{p/s} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (I.3.6)$$

gdzie macierz $\mathbf{L}_{s/p}$ transformacji jest równa:

$$\mathbf{L}_{p/s} = \begin{bmatrix} \cos \phi_{sk} \cos \chi_{sk} + \sin \psi_{sk} \sin \phi_{sk} \sin \chi_{sk} & \cos \phi_{sk} \sin \chi_{sk} - \cos \chi_{sk} \sin \phi_{sk} \sin \psi_{sk} & -\cos \psi_{sk} \sin \phi_{sk} \\ \cos \psi_{sk} \sin \chi_{sk} & \cos \chi_{sk} \cos \psi_{sk} & -\sin \psi_{sk} \\ \cos \chi_{sk} \sin \phi_{sk} - \cos \phi_{sk} \sin \chi_{sk} \sin \psi_{sk} & \sin \phi_{sk} \sin \chi_{sk} + \cos \phi_{sk} \cos \chi_{sk} \sin \psi_{sk} & \cos \phi_{sk} \cos \psi_{sk} \end{bmatrix} \quad (I.3.7)$$

Jeżeli konieczne jest przeliczanie wektora z układu $P\rho\kappa\tau$ do układu $Oxyz$, należy zastosować macierz odwrotną $\mathbf{L}_{s/p} = \mathbf{L}_{p/s}^{-1}$.

Prędkość bezwzględna punktu P względem układu inercyjnego $O_g x_g y_g z_g$ jest równa:

$$\mathbf{V}_P = \mathbf{V} + \mathbf{V}_\Omega \quad (\text{I.3.8})$$

gdzie: \mathbf{V} jest prędkością punktu O (środka masy samolotu) o następujących składowych w układzie $Oxyz$ $\mathbf{V} = [u, v, w]^T$.

\mathbf{V}_Ω jest prędkością wynikającą z ruchu obrotowego układu $Oxyz$ z prędkością kątową Ω . Jest ona określona wyrażeniem:

$$\mathbf{V}_\Omega = \Omega \times \mathbf{r}_P \quad (\text{I.3.9})$$

Ponieważ prędkość kątowa Ω ma w układzie $Oxyz$ składowe $\Omega = [p, q, r]^T$, to uwzględniając, że wektor \mathbf{r}_P określający położenie punktu P w układzie $Oxyz$ ma składowe

$$\mathbf{r}_P = [x_P, y_P, z_P]^T \quad (\text{I.3.10})$$

otrzymuje się:

$$\begin{aligned} (V_\Omega)_x &= qz_P - ry_P \\ (V_\Omega)_y &= rx_P - pz_P \\ (V_\Omega)_z &= py_P - qx_P \end{aligned} \quad (\text{I.3.11})$$

Aby obliczyć siły i momenty aerodynamiczne wytwarzane przez profil skrzydła należy określić jego prędkość względem powietrza \mathbf{V}_{P^*} . Analogicznie do (I.2.5), prędkość ta jest równa różnicy pomiędzy prędkością bezwzględną punktu P \mathbf{V}_P i prędkością wiatru \mathbf{V}_w :

$$\mathbf{V}_{P^*} = \mathbf{V}_P - \mathbf{V}_w \quad (\text{I.3.12})$$

Zatem, uwzględniając zależność (I.3.8), otrzymuje się:

$$\mathbf{V}_{P^*} = \mathbf{V} + \mathbf{V}_\Omega - \mathbf{V}_w \quad (\text{I.3.13})$$

W celu obliczenia obciążeń aerodynamicznych konieczne jest określenie składowych wektora \mathbf{V}_{P^*} w układzie związanym z profilem $P\rho\kappa\tau$. W stosunku do wektorów \mathbf{V} i \mathbf{V}_Ω należy wykorzystać macierz przejścia $\mathbf{L}_{p/s}$. Natomiast w stosunku do wektora prędkości wiatru \mathbf{V}_w konieczne jest przeliczenie jego składowych z układu związanego z Ziemią $O_gx_gy_gz_g$ do układu związanego z samolotem i dalej do układu $P\rho\kappa\tau$. Dzieje się tak dlatego, że wektor ten jest zwykle określany w układzie inercyjnym $O_gx_gy_gz_g$, w którym ma składowe $\mathbf{V}_w = [u_{wg}, v_{wg}, w_{wg}]^T$. Przeliczenia te można zapisać w postaci:

$$(\mathbf{V}_{P^*})_{P\rho\kappa\tau} = \mathbf{L}_{p/s} (\mathbf{V} + \mathbf{V}_\Omega)_{Oxyz} - \mathbf{L}_{p/s} \mathbf{L}_{s/g} (\mathbf{V}_w)_{O_gx_gy_gz_g} \quad (\text{I.3.14})$$

Ostatecznie składowe wektora profilu względem opływającego go powietrza wyliczone w układzie $P\rho\kappa\tau$ są równe:

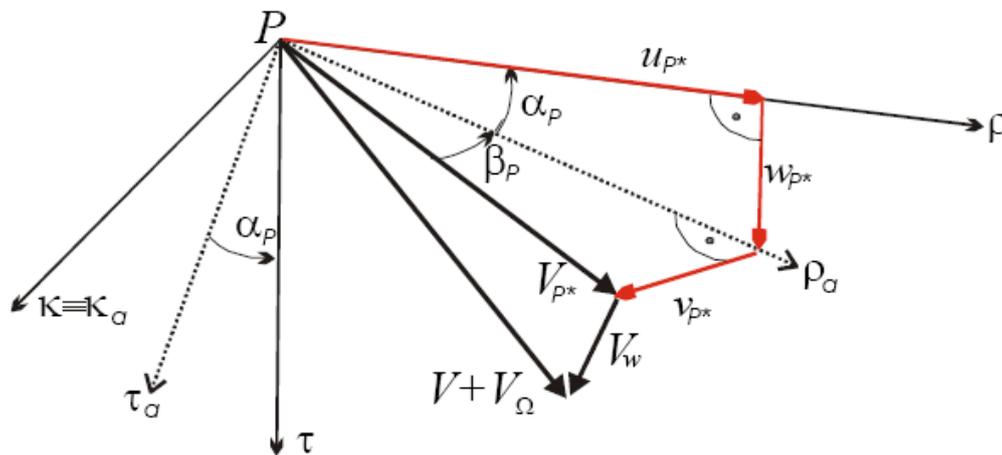
$$\begin{bmatrix} u_{p^*} \\ v_{p^*} \\ w_{p^*} \end{bmatrix} = \mathbf{L}_{p/z} \begin{bmatrix} u + (V_\Omega)_x - u_w \\ v + (V_\Omega)_y - v_w \\ w + (V_\Omega)_z - w_w \end{bmatrix} \quad (\text{I.3.15})$$

gdzie $[u_w, v_w, w_w]^T$ są składowymi prędkości wiatru w układzie $Oxyz$. Są one równe:

$$\begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix} = \mathbf{L}_{z/g} \begin{bmatrix} u_{wg} \\ v_{wg} \\ w_{wg} \end{bmatrix} \quad (\text{I.3.16})$$

Znajomość składowych prędkości profilu względem powietrza u_{p^*} , v_{p^*} , w_{p^*} pozwala obliczyć kąt natarcia α_p profilu pokazany na rysunku I.3.2. Jest on równy:

$$\alpha_p = \arctan \frac{w_{p^*}}{u_{p^*}} \quad (\text{I.3.17})$$



Rys.I.3.2. Określenie kąta natarcia profilu α_p i kąta "ślizgu" β_p

Jednocześnie dodatkowo wykorzystany będzie lokalny układ osi przepływu $P\rho_a\kappa_a\tau_a$. Płaszczyzna $P\rho_a\tau_a$ jest płaszczyzną profilu. Oś $P\rho_a$ skierowana jest wzdłuż rzutu lokalnego wektora prędkości profilu na płaszczyznę $P\rho\tau$. Oś $P\kappa$ pokrywa się z osią $P\kappa_a$. Transformacja pomiędzy układami $P\rho\kappa\tau$ i $P\rho_a\kappa_a\tau_a$ ma postać:

$$\begin{bmatrix} \rho \\ \kappa \\ \tau \end{bmatrix} = \mathbf{L}_{p/pa} \begin{bmatrix} \rho_a \\ \kappa_a \\ \tau_a \end{bmatrix} \quad (\text{I.3.18})$$

gdzie macierz $\mathbf{L}_{p/pa}$ transformacji jest równa:

$$\mathbf{L}_{p/pa} = \begin{bmatrix} \cos \alpha_p & 0 & -\sin \alpha_p \\ 0 & 1 & 0 \\ \sin \alpha_p & 0 & \cos \alpha_p \end{bmatrix} \quad (\text{I.3.19})$$

W układzie $P\rho_a\kappa_a\tau_a$ niezerowe składowe elementarnej siły aerodynamicznej $d\mathbf{R}_{sk}$ powstającej na elemencie skrzydła o długości $d\kappa$ zawierającym rozpatrywany profil są odpowiednio równe³:

$$dR_{\rho_a} = -dP_{xa} = -C_{xa\,pr}(\alpha_p) \frac{\rho V_{p*}^2}{2} dS = -C_{xa\,pr}(\alpha_p) \frac{\rho V_{p*}^2}{2} b(y_p) d\kappa \quad (I.3.20)$$

$$dR_{\tau_a} = -dP_{za} = -C_{za\,pr}(\alpha_p) \frac{\rho V_{p*}^2}{2} dS = -C_{za\,pr}(\alpha_p) \frac{\rho V_{p*}^2}{2} b(y_p) d\kappa \quad (I.3.21)$$

gdzie: dP_{xa} – elementarna siła oporu elementu skrzydła, dP_{za} – elementarna siła nośna elementu skrzydła, dS – powierzchnia elementu skrzydła, ρ – gęstość powietrza, $C_{xa\,pr}$, $C_{za\,pr}$ – współczynniki sił aerodynamicznych profilu określone w rozdziale 1 części II, $b(y_p)$ – cięciwa bieżąca skrzydła, V_{p*} – długość wektora lokalnej prędkości powietrza opływającego profil:

$$V_{p*} = \sqrt{u_{p*}^2 + v_{p*}^2 + w_{p*}^2} \quad (I.3.22)$$

Składowe (I.3.20) i (I.3.21) przelicza się dalej do układu $P\rho\kappa\tau$ wykorzystując zależności:

$$\begin{bmatrix} dR_\rho \\ dR_\kappa \\ dR_\tau \end{bmatrix} = \mathbf{L}_{p/pa} \begin{bmatrix} dR_{\rho_a} \\ 0 \\ dR_{\tau_a} \end{bmatrix} \quad (I.3.23)$$

W wyrażeniach (I.3.20) i (I.3.21) należy uwzględnić wynikający z (I.3.6), przy wykorzystaniu (I.3.5) i (I.3.7), związek:

$$d\kappa = (\mathbf{L}_{p/s})_{21} dx + (\mathbf{L}_{p/s})_{22} dy + (\mathbf{L}_{p/s})_{23} dz = \left[(\mathbf{L}_{p/s})_{21} \frac{dx}{dy} + (\mathbf{L}_{p/s})_{22} + (\mathbf{L}_{p/s})_{23} \frac{dz}{dy} \right] dy \quad (I.3.24)$$

Pochodne dx/dy i dz/dy można wyznaczyć z (I.3.4) i (I.3.5)

Określone w ten sposób siły należy transformować do układu związanego z samolotem wykorzystując macierz $\mathbf{L}_{s/p} = \mathbf{L}_{p/s}^{-1}$:

$$\begin{bmatrix} dR_x \\ dR_y \\ dR_z \end{bmatrix} = \mathbf{L}_{s/p} \begin{bmatrix} dR_\rho \\ dR_\kappa \\ dR_\tau \end{bmatrix} \quad (I.3.25)$$

Siły te generują jednocześnie elementarny moment aerodynamiczny:

$$d\mathbf{M}_{sk} = \mathbf{r}_p \times d\mathbf{R}_{sk} \quad (I.3.26)$$

którego składowe są równe⁴:

³ Przyjęto, że $dR_y = 0$.

⁴ Zgodnie z teorią płaskich przepływów zakłada się, że w kierunku prostopadłym do płaszczyzny profilu siła aerodynamiczna nie powstaje tzn. $dR_{x\alpha} = 0$.

- moment przechylający skrzydła: $dL_{sk} = -dR_z \cdot y_p$ (I.3.27)

- moment pochylający skrzydła: $dM_{sk} = dR_z \cdot x_p - dR_x \cdot z_p$ (I.3.28)

- moment odchylający skrzydła: $dN_{sk} = dR_x \cdot y_p$ (I.3.29)

W wyrażeniach tych x_p oraz y_p zdefiniowane są wyrażeniami (I.3.4) i (I.3.5).

Zgodnie z regułami mechaniki, aerodynamiczny moment pochylający skrzydła (I.3.28) należy uzupełnić o moment względem $1/4$ cięciwy:

$$dM_{1/4} = C_{mpr}(\alpha_p) \frac{\rho V_{P^*}^2}{2} dS = C_{mpr}(\alpha_p) \frac{\rho V_{P^*}^2}{2} b(y_p) d\kappa \quad (I.3.30)$$

Zatem:

$$dM_{sk} = dR_z \cdot x_p - dR_x \cdot z_p + dM_{1/4} \quad (I.3.31)$$

Ostatecznie siły i momenty powstające na skrzydle określa się obliczając następujące całki⁵:

$$R_{xsk} = \int dR_x, \quad R_{ysk} = \int dR_y, \quad R_{zsk} = \int dR_z \quad (I.3.32)$$

$$L_{sk} = \int L_{sk}, \quad M_{sk} = \int dM_{sk}^6, \quad N_{sk} = \int dN_{sk} \quad (I.3.33)$$

Siły (I.3.32) wchodzi w skład prawych stron pierwszych trzech równań układu równań (I.2.35). Uwzględniając je, siły niezależne od prędkości zmiany kąta natarcia α (oznaczone indeksem „*” – patrz rozdział 2.1) oblicza się z zależności:

$$\begin{bmatrix} F_x^* \\ F_y^* \\ F_z^* \end{bmatrix} = \begin{bmatrix} R_{xsk} \\ R_{ysk} \\ R_{zsk} \end{bmatrix} + \mathbf{L}_{z/a} \begin{bmatrix} -C_{xa\text{k-H-V}}^* \frac{\rho V_*^2}{2} S \\ C_{ya\text{k-H-V}}^* \frac{\rho V_*^2}{2} S \\ -C_{za\text{k-H-V}}^* \frac{\rho V_*^2}{2} S \end{bmatrix} + \mathbf{L}_{z/g} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + \begin{bmatrix} P \\ 0 \\ 0 \end{bmatrix} \quad (I.3.34)$$

Zgodnie z (I.3.1) indeksem „k-H-V” oznaczono współczynniki odnoszące się do układu kadłub-usterzenie poziome-usterzenie pionowe”.

Podobnie określono momenty sił występujące po prawych stronach układu (I.2.35):

$$L = L_{sk} + L_{k\text{-H-V}} = L_{sk} + C_{lk\text{-H-V}}^* \frac{\rho V_*^2}{2} Sl$$

$$M^* = M_{sk} + M_{k\text{-H-V}}^* = M_{sk} + C_{mk\text{-H-V}}^* \frac{\rho V_*^2}{2} Sb_a \quad (I.3.35)$$

$$N = N_{sk} + N_{k\text{-H-V}} = N_{sk} + C_{nk\text{-H-V}}^* \frac{\rho V_*^2}{2} Sl$$

⁵ Zgodnie z (I.3.24) zmienną całkowania jest współrzędna y i całkowanie wykonywane jest wzdłuż rozpiętości skrzydła od $-l/2$ do $l/2$, zaś po oderwaniu końcówki skrzydła od $-l/2$ - długość końcówki) do $l/2$.

⁶ W niniejszej pracy moment pochylający samolotu obliczano w sposób inny niż pokazany powyżej. Zastosowana metoda polegała na wykorzystaniu globalnej charakterystyki samolotu $C_m(C_{2a})$. Sposób otrzymania takiej charakterystyki opisano w części II.

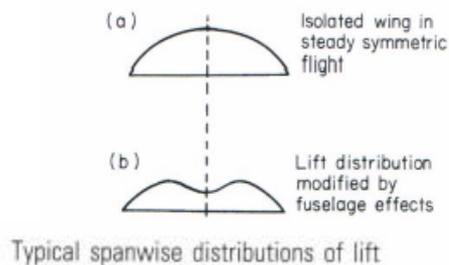
3.3. Wpływ skończonej rozpiętości skrzydła na jego charakterystyki aerodynamiczne

3.3.1. Analiza rozkładu cyrkulacji/współczynnika siły nośnej wzdłuż skrzydła

Obliczenie sił i momentów aerodynamicznych poprzez całkowanie wzdłuż skrzydła, zgodnie z wzorami (I.3.32) i (I.3.33), wymaga uwzględnienia wpływu skończonej rozpiętości na rozkład cyrkulacji wzdłuż skrzydła. W literaturze [3, 8] znaleźć można teoretyczne zależności opisujące ten rozkład dla skrzydła prostokątnego. Jest to rozkład eliptyczny. Ze względu na porosty opis matematyczny będzie on wykorzystany w dalszych obliczeniach. Należy jednak jednoznacznie podkreślić, że skrzydło o innym obrysie wymaga zastosowania bardziej złożonego opisu, co wykracza poza temat niniejszego opracowania. Na podstawie literatury [8] można stwierdzić, że w porównaniu do rozkładu eliptycznego:

- obecność kadłuba zaburza rozkład zmniejszając wartości cyrkulacji lokalnej,
- wychylenie klap zwiększa wartości cyrkulacji lokalnej,
- skos skrzydła deformuje rozkład zwiększając wartość cyrkulacji lokalnej w końcowych przekrojach skrzydła.

Powyższe spostrzeżenia potwierdzają schematyczne rysunki z [8] i [14]. Drugi rysunek dotyczy samolotu Tu-154, co jest szczególnie istotne.



Rys.I.3.3. Wpływ kadłuba na rozkład cyrkulacji wzdłuż skrzydła [8 - rys.5.31]

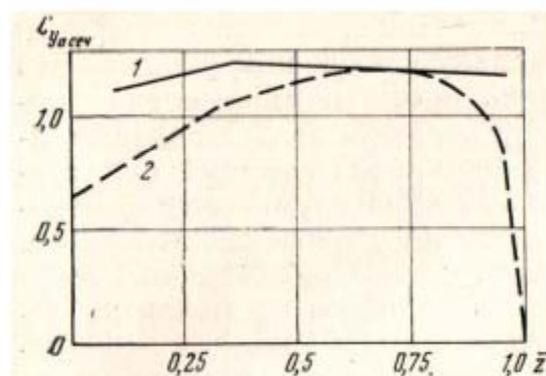


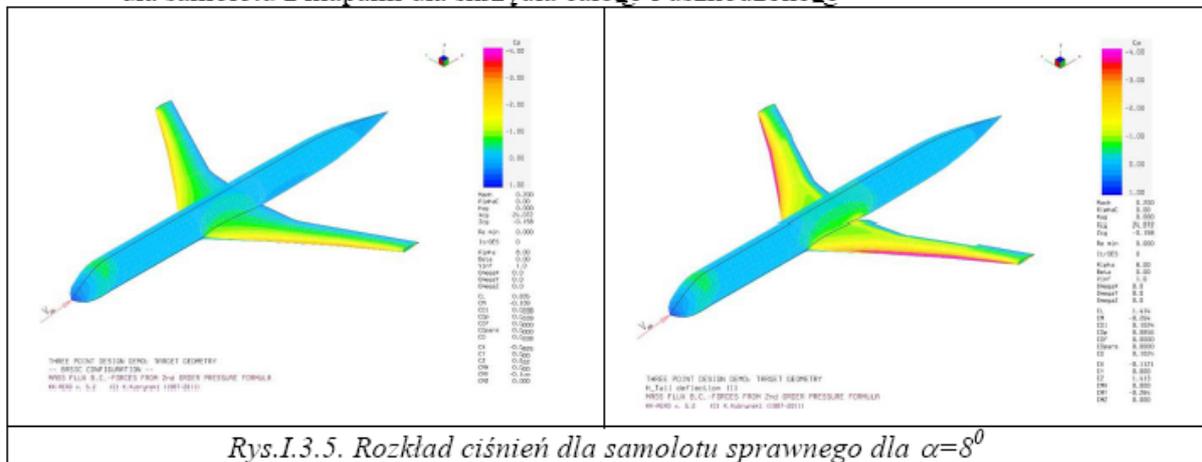
Рис. 8. Схема распределения нагрузки по размаху крыла \bar{z} :

1 — значение коэффициента $C_{y_{aсеч.мах}}$ учетом геометрической и аэродинамической крутки крыла; 2 — значение коэффициента $C_{y_{aсеч}}$ при максимальном (крити-

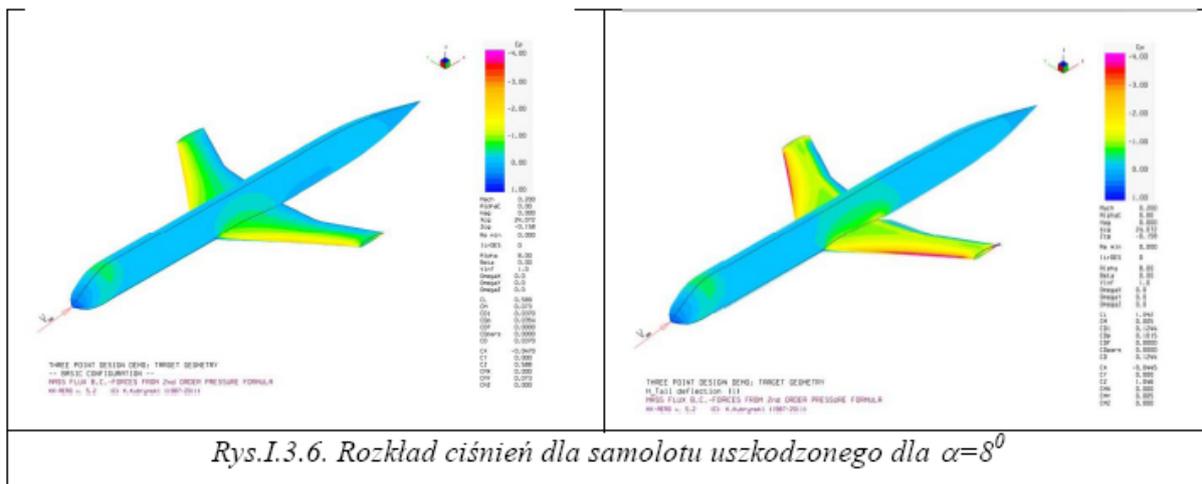
Rys.I.3.4. Rozkład cyrkulacji wzdłuż skrzydła samolotu Tu-154 [14]

Powyższe spostrzeżenia potwierdzone zostały poprzez obliczenia z zakresu mechaniki płynów dedykowane dla samolotu Tu-154. W tym celu odtworzono geometrię kadłuba i skrzydła i wykorzystując metodę panelową określono rozkład współczynnika siły nośnej dla kilku różnych kątów natarcia. Ze względu na programową konieczność zachowania symetrii samolotu dla samolotu z odciętą końcówką skrzydła obliczenia wykonano dla obciętych końcówek obu skrzydeł, co jest widoczne na rysunkach pokazanych poniżej. Pokazano na nich:

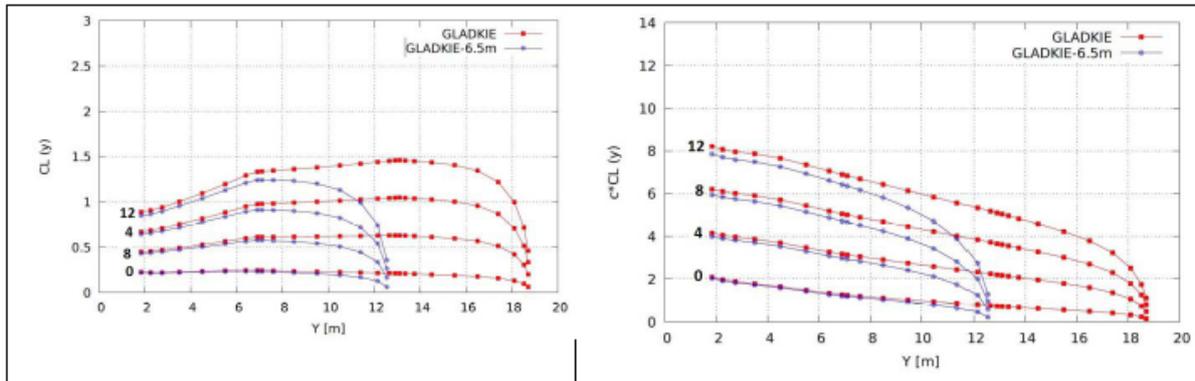
- rozkład ciśnień dla samolotu sprawnego z klapami i bez klap
- rozkład ciśnień dla samolotu z uszkodzonym skrzydłem z klapami i bez klap
- rozkład współczynnika siły nośnej C_{za} i iloczynu bC_{za} dla kątów natarcia $0^{\circ}, 4^{\circ}, 8^{\circ}, 12^{\circ}$ dla samolotu bez klap dla skrzydła całego i uszkodzonego
- rozkład współczynnika siły nośnej C_{za} i iloczynu bC_{za} dla kątów natarcia $0^{\circ}, 4^{\circ}, 8^{\circ}, 12^{\circ}$ dla samolotu z klapami dla skrzydła całego i uszkodzonego



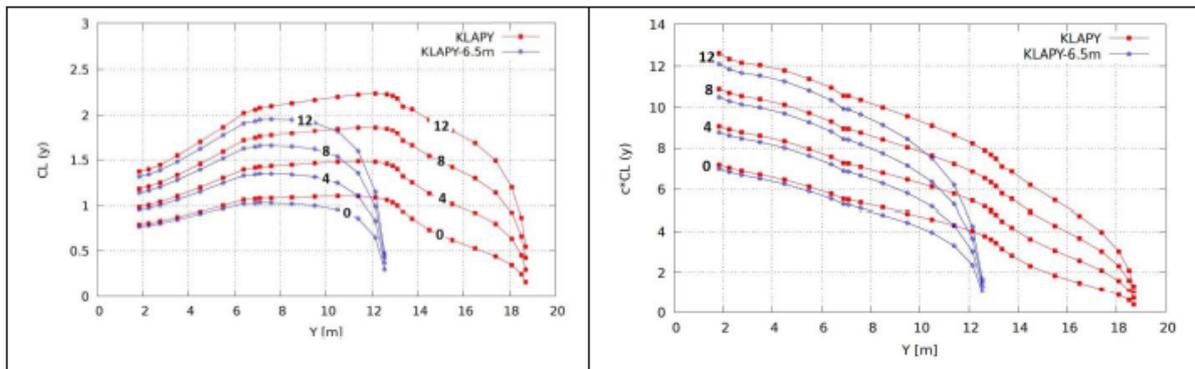
Rys.I.3.5. Rozkład ciśnień dla samolotu sprawnego dla $\alpha=8^{\circ}$



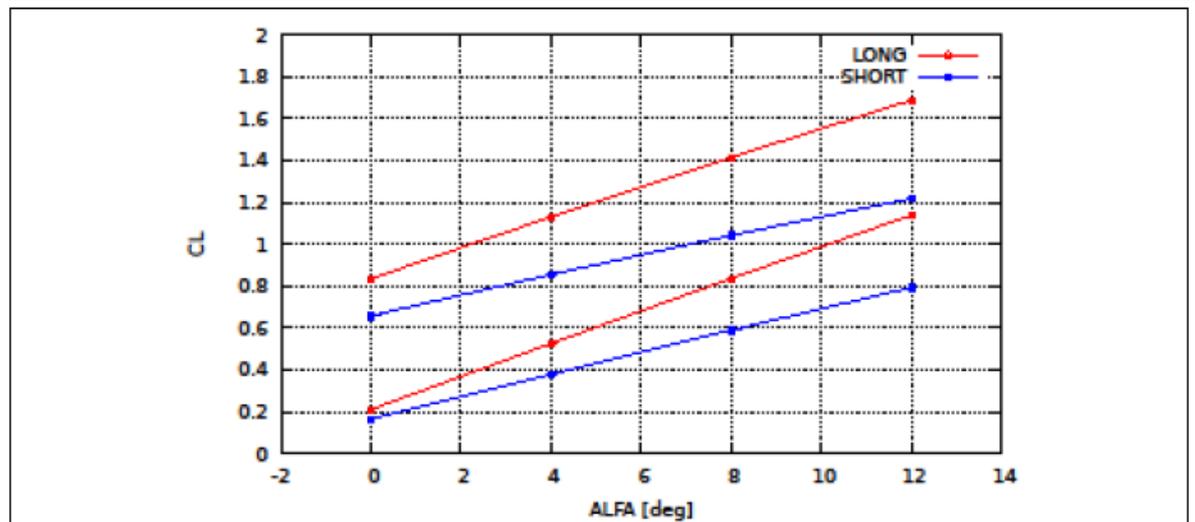
Rys.I.3.6. Rozkład ciśnień dla samolotu uszkodzonego dla $\alpha=8^{\circ}$



Rys.I.3.7. Rozkład C_{za} i iloczynu $b(r) * C_{za}$ wzdłuż skrzydła dla $\alpha=0^{\circ}, 4^{\circ}, 8^{\circ}, 12^{\circ}$ /bez klap/



Rys.I.3.8. Rozkład C_{za} i iloczynu $b(r) * C_{za}$ wzdłuż skrzydła dla $\alpha=0^{\circ}, 4^{\circ}, 8^{\circ}, 12^{\circ}$ /z klapami/



Rys.I.3.9. Zmian współczynnika C_{za} samolotu w wyniku urwania końcówek obu skrzydeł

Pokazane przebiegi są zgodne z przebiegiem dotyczącym samolotu Tu-154 / Rys.I.3.4/. Przeprowadzone obliczenia pozwoliły też oszacować zmianę globalnego współczynnika siły nośnej C_{za} samolotu dla konfiguracji z klapami dla różnych kątów

natarcia. Pokazano to na rysunku I.3.9. Aby ocenić w jaki sposób na wartość C_{za} wpływa urwanie końcówki jednego skrzydła należy wykonać obliczenia, które pokazane zostaną dla kąta 8^0 . Mamy tu:

- dla samolotu sprawnego $C_{za} = 1.4$,

- dla samolotu uszkodzonego /oba skrzydła/ $C_{za} \approx 1.0$.

Różnica wynosi $\Delta C_{za} = 0.4$. Jeżeli urwana zostanie połówka jednego skrzydła to spadek C_{za} będzie równy $\Delta C_{za} = 0.2$. Zatem spadek siły nośnej wynosi $0.2/1.4 = 14\%$. Jeżeli założyc, że siła nośna równoważy ciężar spadek siły nośnej będzie równy $78\text{Ton} = 10.9\text{Tony}$.

Pomimo znacznych różnic pomiędzy pokazanymi powyżej przebiegami rozkładu C_{za} i eliptycznym rozkładem cyrkulacji w obliczeniach zastosowano rozkład eliptyczny. Oznacza to niedoszacowanie sił generowanych na końcach skrzydeł i słabszą niż w rzeczywistości reakcję samolotu na urwanie końcówki skrzydła.

3.3.2. Analiza rozkładu cyrkulacji/współczynnika siły nośnej wzdłuż skrzydła

Eliptyczny rozkład cyrkulacji opisany jest równaniem:

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \frac{y^2}{\left(\frac{l}{2}\right)^2}} \quad (\text{I.3.36})$$

Siła nośna przypadająca na jednostkę rozpiętości skrzydła jest określona wzorem Kutta-Żukowskiego:

$$\frac{dP_{za}}{dy} = \rho V \Gamma(y) \quad (\text{I.3.37})$$

Zatem całkowita siła nośna skrzydła jest równa:

$$dP_{za} = \int_{-l/2}^{l/2} \rho V \Gamma(y) dy = \frac{\rho V \Gamma_0}{l} \frac{\pi}{4} \quad (\text{I.3.38})$$

Porównując ten wzór z formułą (I.2.3) mamy:

$$\Gamma_0 = \frac{2C_{za}^* V S}{\pi l} \quad 7 \quad (\text{I.3.39})$$

Siła nośna powstająca na elemencie skrzydła o długości dy zgodnie z (I.3.37) jest równa:

$$dP_{za} = \rho V \Gamma(y) dy \quad (\text{I.3.40})$$

Jednocześnie mamy:

⁷ Dla odróżnienia globalny współczynnik siły nośnej samolotu oznaczono tu C_{za}^* . Współczynnik ten jest odczytywany z charakterystyki $C_{za}(a)$ określonej dla całego samolotu. Pokazane to będzie w części II.

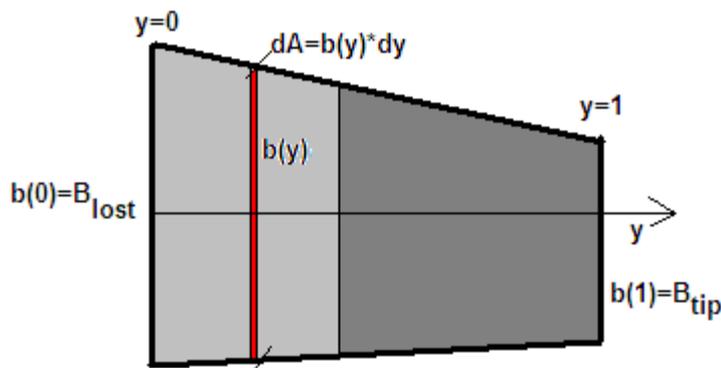
$$dP_{za} = C_{za}(y) \frac{\rho V^2}{2} dS = C_{za}(y) \frac{\rho V^2}{2} b dy \quad (I.3.41)$$

Z porównania otrzymujemy:

$$C_{za}(y) = \frac{2}{Vb} \Gamma(y) = \frac{4C_{za}^* S}{\pi b(y)} \sqrt{1 - \frac{y^2}{\left(\frac{l}{2}\right)^2}} \quad (I.3.42)$$

Wyrażenie to było wykorzystywane do oszacowania współczynnika siły nośnej w poszczególnych przekrojach skrzydła.

APPENDIX 6. Simple Area Method.



Contribution to the moment about x0 line is
 $dM = dF \cdot (y - y_t) = CL(y) \cdot b(y) \cdot (y - y_t) \cdot dy$ for $CL(y) = CL$
 $dMt = dM / CL = b(y) \cdot (y - y_t) \cdot dy$

Geometrical data of wing section 1

$B_0 := 7.445\text{-m}$	Chord of main wing section at $y=0$	
$B_t := 2.138\text{-m}$	Chord of main wing section at tip, $y=L_{hs}$	
$L_{hs} := 18.775\text{-m}$	Length of halfspan	
$L_{lost} := 5.54\text{-m}$	Lost tip length including "birch tree damage width"	
$L_{LW} := L_{hs} - L_{lost}$	Length of remaining wing section	$L_{LW} = 13.235\text{ m}$

The chord length at the new wing tip can be found as

$$B_{lost}(L) := \frac{B_0 - B_t}{L_{hs}} \cdot L + B_t \quad B_{lost}(L_{lost}) = 3.704\text{ m}$$

The area lost can be found as:

$$A_{lost}(L) := \frac{A_{wing}}{2} - A_{total}(L_{hs} - L) \quad A_{lost}(L_{lost}) = 16.19\text{ m}^2$$

The length fraction of the lost wing tip can be found as: $\frac{L_{lost}}{L_{hs}} = 0.295$

The chord length as a function of the local parameter x:

$$\Delta B(L) := B_{\text{lost}}(L) - B_t \quad \text{Chord change over wing tip} \quad \Delta B(L_{\text{lost}}) = 1.566 \text{ m}$$

$$b(y, L) := B_{\text{lost}}(L) - \Delta B(L) \cdot y \quad \text{Chord function of the local wingtip parameter } y \text{ (see fig).}$$

Assuming near to constant local lift coefficients $CL(y)$ the distance $y=yt$ can be found, where the force integral on each side of $y=yt$ balance (aero dynamical force centre):

$$yt := 0.457 \quad \text{results in } Mt(xt)=0 \quad \text{where}$$

$$Mt(yt, L) := \int_0^1 cx(L_{hs} - y \cdot L) \cdot [y - (1 - yt)] dy \quad Mt(yt, L_{\text{lost}}) = 0 \text{ m}$$

The distance from the centre of the plane to the lift centre of the missing wing tip :

$$L_c := L_{hs} - (1 - yt) \cdot L_{\text{lost}} \quad L_c = 15.767 \text{ m}$$

The distance yt resulting in the moment on each side of $y=yt$ can be found by iteration:

$$yt(L) := \left| \begin{array}{l} ga \leftarrow 0.5 \\ Mg \leftarrow \frac{Mt(ga, L)}{m} \\ \text{while } |Mg| > 0.005 \\ \quad \left| \begin{array}{l} ga \leftarrow ga - Mg \cdot 0.001 \\ Mg \leftarrow \frac{Mt(ga, L)}{m} \end{array} \right. \\ ga \end{array} \right.$$

$$yt(L_{\text{lost}}) = 0.457$$

$$xl := 0 \text{ m}, 0.1 \text{ m} \dots L_{hs}$$

$$A_{\text{wing}} := 180 \cdot \text{m}^2$$

Total wing area

$$C_s(L) := \frac{A_{\text{wing}} - A_{\text{lost}}(L)}{A_{\text{wing}}}$$

Simple lift coefficient proportional to wing area

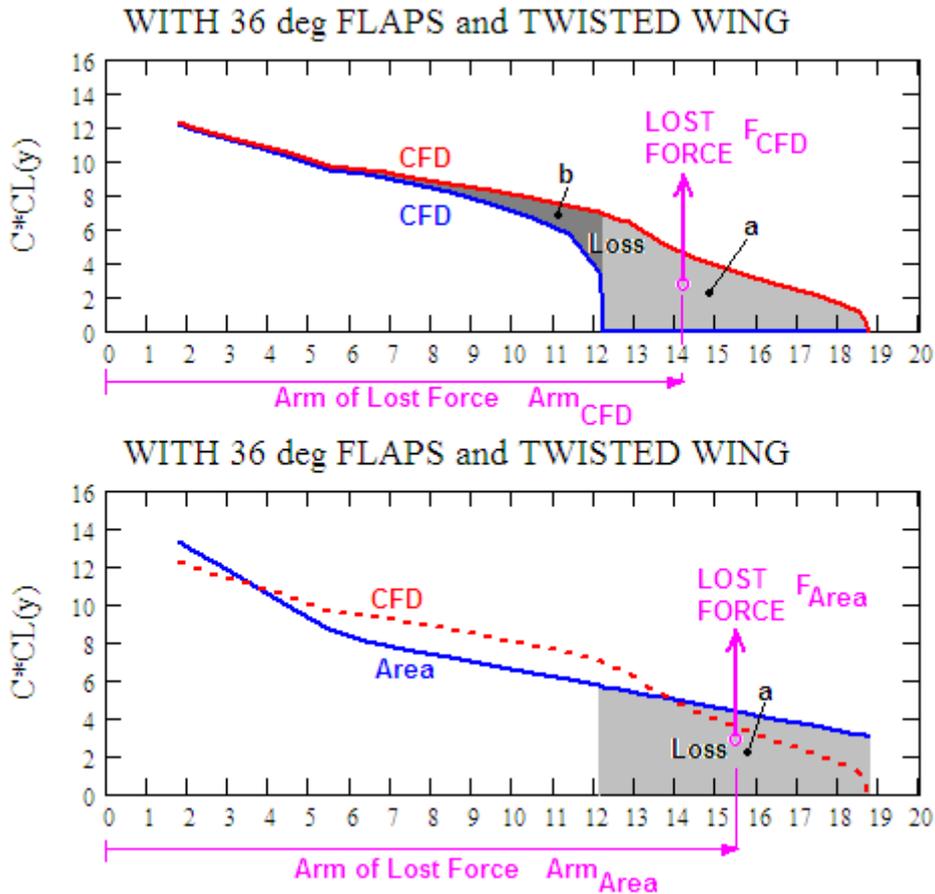
$$L_w(L) := L_{hs} - L$$

Length of remaining wing section

$$L_s(L) := L_{hs} - (1 - yt(L)) \cdot L$$

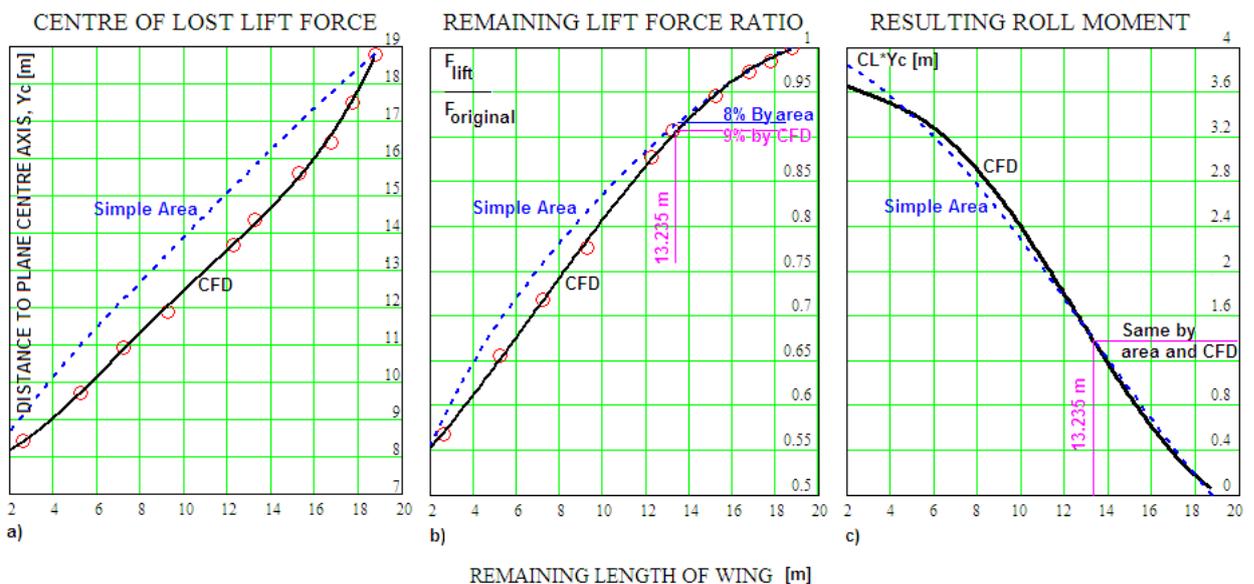
Centre of lost lift force (ARM)

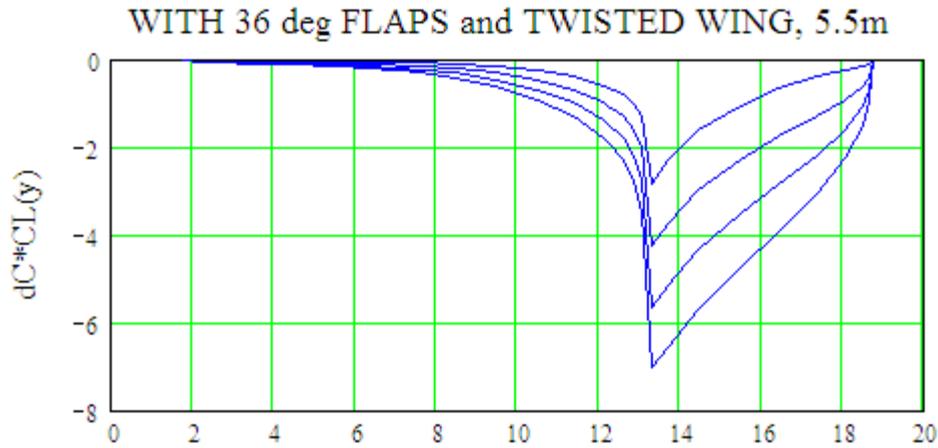




The loss by CFD/CFD (upper fig) and by CFD/Area (lower fig). The part a is the loss carried by the lost wing area, and the part b is the loss associated with the change in the local lift coefficients of the remaining wing area. $Arm_{CFD} < Arm_{Area}$ and $F_{CFD} > F_{Area}$, but it turns out that

$$M_{CFD} = Arm_{CFD} * F_{CFD} \approx M_{Area} = Arm_{Area} * F_{Area}$$





The change in $C^*CL(y)$ for $\alpha_{gk} = 0^\circ, 4^\circ, 8^\circ$ and 12° . The integral of $dC^*CL(y)$ equals the loss in lifting force, the aerodynamic centre, Y_c , is found as:

$$Y_c = \frac{\int_0^{L_{hs}} cx(y) \cdot (CL(y) - CLb(y)) \cdot y \, dy}{\int_0^{L_{hs}} cx(y) \cdot (CL(y) - CLb(y)) \, dy}$$

Where

- $cx(y)$: is the chord length
- $CL(y)$: is the local lifting coefficient function of the intact wing at α of interest
- $CLb(y)$: is the local lifting coefficient function of the broken wing at α of interest
- y : is the span coordinate

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